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Decomposability of Normal Projective Curvature Tensor in Finsler Spaces

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Abstract: **The decomposition of curvature tensor field was studied by K.Takano [4]. The decomposability of curvature tensor in Finsler manifold was studied by P.N, Pandey [2].**
The purpose of the present paper is to decompose the normal projective curvature tensor N^i_{jkh} in recurrent Finsler space and study the properties of conformal decomposition tensor.

Keywords: **Finsler space, normal projective curvature tensor, recurrent Finsler space.**

1. INTRODUCTION

We consider an $n-$ dimensional Finsler space F_n in which the Normal projective curvature tensor defined by [1]

(1.1) $$N^i_{jkh} = H^i_{jkh} - \frac{1}{n+1} y^i \dot\partial_j H^r_{rkh} .$$

Contracting the indices i and h in equation (1.1) and using the fact that the tensor H^r_{rkh} is positively homogeneous of degree zero in y^i , we have

(1.2) $$N^r_{rkh} = H^r_{rkh} .$$

Transvecting (1.1) by y^j and using $H^i_{jkh}\, y^j = H^i_{kh}$, we get

(1.3) $$N^i_{jkh}\, y^j = H^i_{kh} .$$

The normal projective curvature tensor is skew-symmetric in its last two lower indices ,i.e.

(1.4) $$N^i_{jkh} = -N^i_{jhk} .$$

The normal projective curvature tensor satisfies the identities

(1.5) $$N^i_{jkh} + N^i_{khj} + N^i_{hjk} = 0 .$$

(1.6) $$\lambda_n N^i_{jkh} + \lambda_k N^i_{jhn} + \lambda_h N^i_{jnk} = 0 .$$

Contracting the indices i and h in equation (1.1), we get

(1.7) $$N_{jk} = H_{jk} - \frac{1}{n+1} y^i \dot\partial_j H^r_{rki} .$$

Shalini Dikshit [1] defined a Finsler space in which the normal projective curvature tensor is recurrent, i.e.

(1.8) $$\mathcal{B}_n N^i_{jkh} = \lambda_n N^i_{jkh} , N^i_{jkh} \neq 0 .$$

She also defined a Finsler space in which the normal projective curvature tensor is birecurrent ,i.e.

(1.9) $$\mathcal{B}_m \mathcal{B}_n N^i_{jkh} = a_{mn} N^i_{jkh} .$$

2. DECOMPOSITION OF NORMAL PROJECTIVE CURVATURE TENSOR IN FINSLER SPACE

Let us consider the normal projective tensor N^i_{jkh} in the form

(2.1) $$N^i_{jkh} = X^i_j \, y_{kh} \, .$$

where X^i_j is non-zero tensor and y_{kh} is skew symmetric decomposition tensor .

The space equipped with such decomposition of normal projective curvature tensor in recurrent Finsler space and denote it by $R - F_n$.

Differentiating (2.1) covariantly with respect to x^n in the sense of Berwald, we get

(2.2) $$\mathcal{B}_n N^i_{jkh} = \mathcal{B}_n X^i_j \, y_{kh} + X^i_j \mathcal{B}_n \, y_{kh} \, .$$

Using (1.8) in (2.2), we get

(2.3) $$\lambda_n N^i_{jkh} = \mu_n X^i_j \, y_{kh} + X^i_j \mathcal{B}_n \, y_{kh},$$

where

(2.4) $$\mathcal{B}_n X^i_j = \mu_n X^i_j \, .$$

From equation (2.1) and equation (2.3), we get

$$\lambda_n X^i_j \, y_{kh} = \mu_n X^i_j \, y_{kh} + X^i_j \mathcal{B}_n \, y_{kh} \, ,$$

or

(2.5) $$\mathcal{B}_n \, y_{kh} = (\lambda_n - \mu_n) \, y_{kh} \, .$$

Let us assume that $\lambda_n \neq \mu_n$, the equation (2.5) may be written as

(2.6) $$\mathcal{B}_n \, y_{kh} = v_n \, y_{kh} \, .$$

where

(2.7) $$v_n = (\lambda_n - \mu_n) \, .$$

If the above equation (2.6) is true then (2.3) yield

$$\lambda_n N^i_{jkh} = \mu_n X^i_j \, y_{kh} + X^i_j v_n \, y_{kh} \, .$$

or

$$\lambda_n X^i_j \, y_{kh} = \mu_n X^i_j \, y_{kh} + X^i_j v_n \, y_{kh} \, .$$

or

(2.8) $$\lambda_n \, y_{kh} = (\mu_n + v_n) \, y_{kh} \, .$$

Thus, we have

Theorem2.1. In $NPR - F_n$, the necessary and sufficient condition for the decomposition tensor to be recurrent is that the recurrence vector λ_n is not equal to recurrence vector μ_n .

Let us assume that the recurrence vector λ_n is equal to recurrence vector μ_n such that

(2.9) $$\lambda_n = \mu_n \, .$$

In view of (2.9) and (2.6) immediately reduces to

(2.10) $$\mathcal{B}_n \, y_{kh} = 0 \, .$$

Using (2.10) in (2.2), we have

$$\mathcal{B}_n N^i_{jkh} = \mathcal{B}_n X^i_j \, y_{kh} \, .$$

or

(2.11) $\mathcal{B}_n N^i_{jkh} = \mu_n X^i_j \, y_{kh}$.

Adding the expressions obtained by cyclic change of (2.11) with respect to the indices k, h and m, we have

(2.12) $\mathcal{B}_n N^i_{jkh} + \mathcal{B}_k N^i_{jhn} + \mathcal{B}_h N^i_{jnk} = X^i_j(\mu_n \, y_{kh} + \mu_k \, y_{hn} + \mu_h \, y_{nk})$.

In view of (1.6) , equation (2.12) reduces to

(2.13) $X^i_j(\mu_n \, y_{kh} + \mu_k \, y_{hn} + \mu_h \, y_{nk}) = 0$.

Since X^i_j is non-zero tensor it implies

$(\mu_n \, y_{kh} + \mu_k \, y_{hn} + \mu_h \, y_{nk}) = 0$.

or

(2.14) $(\lambda_n \, y_{kh} + \lambda_k \, y_{hn} + \lambda_h \, y_{nk}) = 0$.

Theorem2.2. In $NPR - F_n$, under the decomposition (2.1), if the vector λ_n is equal to μ_n, the decomposition tensor satisfies the identity (2.14).

Differentiating (2.11) covariantly with respect to x^m in the sense of Berwald and using (2.10), we get

(2.15) $\mathcal{B}_m \, \mathcal{B}_n N^i_{jkh} = \mathcal{B}_m(\mu_n X^i_j) \, y_{kh} = [(\mathcal{B}_m \mu_n) X^i_j + \mu_n(\mathcal{B}_m X^i_j)] \, y_{kh}$.

In view of (2.4) the above equation may be written as

(2.16) $\mathcal{B}_m \, \mathcal{B}_n N^i_{jkh} = (\mathcal{B}_m \mu_n + \mu_n \mu_m) X^i_j \, y_{kh}$.

Using (1.9) and (2.1), we get

(2.17) $a_{mn} X^i_j \, y_{kh} = (\mathcal{B}_m \mu_n + \mu_n \mu_m) X^i_j \, y_{kh}$.

From (2.16) and (2.17), we have

(2.18) $a_{mn} = \mathcal{B}_m \mu_n + \mu_n \mu_m$.

Thus, we conclude that

Theorem2.3. . In $NPR - F_n$, under the decomposition (2.1), if the recurrence vector λ_n is equal to recurrence vector μ_n, for which recurrence vector field μ_n satisfies the condition $(\mathcal{B}_m \mu_n + \mu_n \mu_m) \neq 0$.

Interchanging the indices m and n in (2.16) and subtracting the equation which obtained to (2.16), we have

(2.19) $\mathcal{B}_n \, \mathcal{B}_m N^i_{jkh} - \mathcal{B}_m \, \mathcal{B}_n N^i_{jkh} = (\mathcal{B}_n \mu_m - \mathcal{B}_m \mu_n) X^i_j \, y_{kh}$.

Accordingly we state

Corollary2.1. In $NPR - F_n$, under the decomposition (2.1), if the recurrence vector λ_n is equal to recurrence vector μ_n, the normal projective curvature tensor satisfies the identity (2.19).

Differentiating (2.6) covariantly with respect to x^m in the sense of Berwald, we get

(2.20) $\mathcal{B}_m \, \mathcal{B}_n \, y_{kh} = (\mathcal{B}_m v_n) \, y_{kh} + v_n \mathcal{B}_m \, y_{kh}$

$= (\mathcal{B}_m \lambda_n - \mathcal{B}_m \mu_n) \, y_{kh} + (\lambda_n - \mu_n) \mathcal{B}_m \, y_{kh}$.

In view of (2.6), the equation (2.20) may be written as

(2.21) $\mathcal{B}_m \, \mathcal{B}_n \, y_{kh} = (\mathcal{B}_m \lambda_n - \mathcal{B}_m \mu_n) \, y_{kh} + (\lambda_n - \mu_n) v_m \, y_{kh}$.

or

(2.22) $\mathcal{B}_m \, \mathcal{B}_n \, y_{kh} = (\mathcal{B}_m \lambda_n - \mathcal{B}_m \mu_n) \, y_{kh} + (\lambda_n - \mu_n)(\lambda_m - \mu_m) \, y_{kh}$.

or

(2.23) $\mathcal{B}_m \mathcal{B}_n y_{kh} = (\mathcal{B}_m\lambda_n - \mathcal{B}_m\mu_n + \lambda_n\lambda_m - \lambda_n\mu_m$
$-\mu_n\lambda_m + \mu_n\mu_m) y_{kh}$.

Theorem2.4. In $NPR - F_n$, under the decomposition (2.1), the second order covariant derivative of decomposition tensor y_{kh} satisfies the relation(2.23).

In view of (2.9), equation (2.23) immediately reduces to

(2.24) $\mathcal{B}_m \mathcal{B}_n y_{kh} = 0$.

Corollary2.2. In $NPR - F_n$, the second order covariant derivative of decomposition tensor y_{kh} vanish , if the vector λ_n is equal to μ_n .

Differentiating (2.4) covariantly with respect to x^m in the sense of Berwald, we get

(2.25) $\mathcal{B}_m \mathcal{B}_n X_j^i = (\mathcal{B}_m\mu_n)X_j^i + \mu_n\mathcal{B}_m X_j^i$.

Using equation (2.4) in equation (2.25), we get

(2.26) $\mathcal{B}_m \mathcal{B}_n X_j^i = (\mathcal{B}_m\mu_n)X_j^i + \mu_n\mu_m X_j^i = (\mathcal{B}_m\mu_n + \mu_n\mu_m)X_j^i$.

Theorem2.5. In $NPR - F_n$, under the decomposition (2.1), the second order covariant derivative of the tensor X_j^i satisfies the relation(2.26).

REFERENCES

[1] Dikshit, S.: Certain types of recurrences in Finsler spaces, D. phil. Thesis, University of Allahabad, India , 1992 .

[2] Pandey, P.N.: On decomposability of curvature tensor of a Finsler manifold Acta Mathmatica, Academia Scieniarum Hungaricae , Tomus 38(1-4) ,1981, 109-116.

[3] Rund, H .: The differential geometry of Finsler space, Springerverlag, Berlin Gottingen- Heidelberg, (1959) ,2nd edit. (in Russian), Nauka,(Moscow),(1981) .

[4] Takano, K: Affine motion in non Riemannian K*-space I, and II(with M. Okumura),IV and V, Tensor, N.S.,11,137-143, 161-173, 174-181, 245-253, 270-278.

[5] Thakur, M. Mishra.C.K and Lodhi, G.: Decomposability of projective curvature tensor in recurrent Finsler space($WR - F_n$), International J. of computer Applications , V(100), No.19, 2014, 32-34.