

PAPR Analysis and Channel Estimation Techniques for 3GPP LTE System

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Abstract: High data rates and secured data communication has become an unavoidable need of every mobile users. 3G technology provided greater data speed and secured networks compared to its predecessor 2G or 2.5G. The highest bit rates in commercially deployed wireless systems are achieved by means of Orthogonal Frequency Division Multiplexing (OFDM). The next advance in cellular systems, under investigation by Third Generation Partnership Project (3GPP), also anticipates the adoption of OFDMA to achieve high data rates. But a modified form of OFDMA i.e. SCFDMA (Single Carrier FDMA) having similar throughput performance and essentially the same complexity has been implemented as it has an edge over OFDMA having lower PAPR (peak to average power ratio). SCFDMA is currently a strong candidate for the uplink multiple access in the Long Term Evolution of cellular systems under consideration by the 3GPP. In our project we have worked on PAPR analysis of OFDMA, SCFDMA and various other SCFDMA (with different subcarrier mapping). Though SCFDMA had larger ISI it has lower PAPR which help in avoiding the need of an inefficient linear power amplifier. We analyzed various modulation techniques and implemented various kinds of pulse shaping filters and compared the PAPR for IFDMA, DFDMA and LFDMA (kinds of SCFDMA). The PAPR Performance of IFDMA was found to be better than LFDMA and DFDMA. Like other communication systems, in SCFDMA we encounter many trade-offs between design parameters (such as roll-off factor) and performance.

The project report also constitutes the channel estimation techniques implemented in OFDM systems. Due to multipath fading the channel impulse response fluctuates for different subcarriers in different time slots. So we need to estimate the channel impulse response at various times to extract error free data. As we have taken a slow Rayleigh fading channel in our study we used block type pilot arrangement channel estimation which use LS (least square), MMSE (minimum mean square error) estimator. Due to higher complexity of the MMSE estimator, modified MMSE is implemented where we had to do the tradeoff with the performance. There are various other adaptive estimation techniques like LMS and RLS for estimating blind channels and comb type pilot arrangement estimation techniques for fast fading channels.

Keywords: OFDMA, SCFDMA, PAPR, cyclic prefix, LS, MMSE, MSE, SER, SNR.

I. INTRODUCTION

First mobile radio telephone (car mounted) was made in 1924 by AT & T Bell Laboratories in 1924. Since then the mobile phone industry has taken a hectic growth and it promises to deliver more in 21st century. The Bell Labs introduced cellular concept in 1960s which opened up path for public communication service in large scale. With the development of highly reliable, miniature, solid-state radio frequency hardware in the 1970s, the wireless communication era was born. Shown below is the overview of technology transformation from 1G to 3G.

1.1 Evolution of 3G

a) 1G (early 1980s):

- Analog speech communication
- Analog FDMA/FDD
- Ex-AMPS standard by Bell Labs

b) 2G (early 1990s):

- Digital speech communication
- Handoff, more secure communication
- TDMA and CDMA schemes
- Ex-Four major standards
 - GSM
 - IS-136/IS-54 NADC,PDC(Japan)
 - IS-95 cdmaOne

c) 2.5G (Mid 1990s):

- Improvement of data rate
- Upgradation of 2G
- Ex-HSCSD,GPRS,EDGE(from GSM)
IS-95B (from cdmaOne)

d) 3G (late 1990s):

- A global standard for communication
- High data rate
- Ex-WCDMA (UMTS), cdma2000, TD-SCDMA

The International Telecommunications Union (ITU) formulated a plan to implement a global frequency band in the 2000 Mhz range that would support a single and common wireless communication standard for all countries throughout the world. This plan renamed International Mobile Telephone (IMT-2000) has been successful to develop many standards for future implementation but the world is divided between two camps: TDMA standard and CDMA standard. So IMT-2000 group is divided into two camps: 3GPP (3G partnership project having backward compatibility with GSM/IS-136/PDC) and 3GPP2 (3G partnership project for cdma2000 having backward compatibility with CDMA).

The 3GPP LTE (Long Term Evolution) was a recent standard introduced by 3GPP group which promises high-speed data, multimedia unicast and multimedia broadcast services. The specifications include the following:

Multiple Access Schemes:

DL: OFDMA with CP

UL: SCFDMA with CP

Modulation:

UL/DL: QPSK, 16QAM, 64QAM

Coding:

Convolutional code, Rel-6 Turbo code

1.2 Multi Carrier Modulations

Unlike single carrier systems, OFDM communication systems do not rely on increased symbol rates in order to achieve higher data rates. OFDM is a multicarrier digital modulation scheme .OFDM systems break the available bandwidth into many narrower sub-carriers and transmit the data in parallel streams. Each subcarrier is modulated using varying levels of QAM modulation, e.g. QPSK, QAM, 64QAM or possibly higher orders depending on signal quality. Each OFDM symbol

is therefore a linear combination of the instantaneous signals on each of the sub-carriers in the channel. This scheme facilitates efficient use of bandwidth and reduced Inter Symbol Interference (ISI). But another problem is high Peak to Average Power Ratio (PAPR) OFDM symbols. To counter this we use a modified scheme called Single Carrier FDMA (SC-FDMA). The advantages are reduced PAPR and frequency domain equalization.

1.3 Estimating the channel

The mobile radio channel sharply influences the performance of wireless communication system and needs rigorous analysis before modeling any system. The transmission path between the transmitter and the receiver can be a simple line of sight to one the severely faded by trees, buildings and mountains. Radio channels are random and time varying in nature, so don't offer an easy analysis. So modeling the radio channel taking all the influencing parameters into account is a tough task. So estimating the channel by using its response to a known data and repeating this process time and again, there by using channel statistics. Various estimation techniques are used in literature, here we have exploited the use of LS, MMSE and Modified MMSE estimator.

1.4 Objective and Outline of Thesis

The main objectives of thesis are: (1) A comparative study of OFDMA and SCFDMA which are used for downlink and uplink communication in 3GPP LTE respectively. PAPR analysis for both the techniques under different conditions. (2) Analysis of different kinds of channel estimation techniques used in OFDM, their principles and performance in terms of Mean Squared Error and Symbol Error rate.

The thesis is organized as follows:

Chapter 2 gives the characteristics of mobile radio channels and different ways to model channel impulse responses.

Chapter 3 discusses about the basics of OFDM and SCFDMA. Then a comparative study using PAPR analysis and preference of SCFDMA for uplink in 3GPP LTE.

Chapter 4 investigates different channel estimation techniques, they are LS estimator, LMMSE estimator and modified MMSE estimator. Algorithms of each with analysis of performance is given in this chapter

Chapter 5 deals with simulations and results under different parametric conditions.

Chapter 6 concludes on the entire discussion.

II. CHARACTERISTICS OF MOBILE RADIO CHANNEL

2.1 Introduction

A channel ideally should contain only one copy of transmitted signal coming in the line of Sight path from transmitter to receiver, then there would a perfect reconstruction of original signal. But in reality this doesn't happen. Rather the received signal consists of a combination of attenuated, reflected, refracted and diffracted replicas of original signal. So the channel gets faded both in time and frequency domain. Also the channel adds noise to the signal which further complicates the procedure. If there's relative motion in the channel then frequency shift occurs (Doppler Effect). Knowledge of all these phenomena is necessary in order to model the channel for radio wave propagation.

2.2 Types of Fading

The propagation model mainly focuses on predicting the average received signal strength at a given T-R (Transmitter-Receiver) separation and radial variation for the specified separation. So we can classify fading into two types: Large-scale fading and Small scale fading. Large scale fading attributes for variation in signal strength over large T-R separation distances. Large scale models try to find out mean signal power attenuation or path loss due motion over large area around transmitter or receiver. Small scale fading characterize rapid fluctuation of received signal strength over short T-R separations and for short period of time. So the signal is a sum of many signals coming from different directions with different attenuation which brings dramatic changes in signal amplitude and phase.

Various models exist in literature for large scale fading. They are like empirical models such as Okumura model, Hata model, cost 136 model etc; indoor models like Log-distance path loss model, Ericsson multiple breakpoint model, Attenuation factor model etc. Large scale fading models find applications in wireless network planning for an area and modeling path loss over a large distance. So, large scale path loss models are more important for cell site planning but less for communication system design. So we will next discuss small scale fading in a little detail.

2.3 Small-scale Fading

Fading is caused by interference between two or more forms of transmitted signal that arrive at receiver at slightly different times. These components are called multipath components. The complete set of multi paths has to be known for modeling the multipath channel. Each path is characterized by three parameters namely delay, attenuation and phase shift.

The discrete time variant channel impulse response of the multipath channel is given by $h(\tau, t)$

$$h(\tau, t) = \sum_m \alpha_m(t) e^{-j2\pi f_c \tau_m(t)} \delta(t - \tau_m(t))$$

where,

$\alpha_m(t)$ is the attenuation in the m^{th} path at time t

$\tau_m(t)$ is the propagation delay in the m^{th} path at time t

$e^{-j2\pi f_c \tau_m(t)}$ is the phase shift for carrier frequency f_c for m^{th} multipath component

$\delta(\cdot)$ is the dirac delta function

The above model takes into all the modifications that a multipath channel can make to the signal.

2.4 Critical Channel Parameters

Two kinds of spreading occur when a signal passes through a channel. They are

- Multipath delay spread
- Doppler spread

Multipath delay spread occurs because of time dispersive nature of the channel in local area. Because delayed versions of original signal is superimposed at the receiver so the received signal spreads in time domain or shows time dispersion. Parameter used to describe this is rms delay spread denoted by σ_τ . This is defined as the standard deviation value of the delay weighed proportional to the energy of waves. Coherence bandwidth (f_0) is analogous to delay spread used to characterize the channel in frequency domain. It's the statistical measure of the range of frequencies for which all components are passed with equal gain and linear phase. So we can say

$$f_0 \propto \frac{1}{\sigma_\tau}$$

Doppler spread occurs because of relative motion between transmitter and receiver or motion of objects in the channel. So it occurs because of time variance nature of the channel. Because of relative motion Doppler shift of frequency occurs which broadens the signal in frequency domain or shows frequency dispersion. Parameter used to characterize this is Doppler spread denoted by f_d . This is defined as the range of frequencies over which the Doppler spectrum is non-zero. Coherence time (T_c) is the time domain dual of Doppler spread and used to characterize the time varying nature of the frequency dispersiveness of channel. It's the statistical measure of time duration over which the channel impulse response is essentially constant. So that we can write

$$T_c \propto \frac{1}{f_d}$$

2.5 Types of Small-scale Fading

Small-scale fading occurs due to two propagation mechanisms as described above. They are

Due to multipath delay spread

- Flat fading
- Frequency selective fading Due to Doppler spread
- Fast fading
- Slow fading

If the bandwidth of the channel is less than range of frequency over which the channel has constant gain and linear phase, then the signal undergoes flat fading. This type of fading is common in literature as this is analogous to a low pass filter. After passing the spectral characteristics of the channel remains unchanged but the gain changes with time. So in terms of channel parameters

If $f_m < f_0$ and $T_s > \sigma_\tau$ where f_m : signal bandwidth and T_s : symbol period

Then the channel creates flat fading

If the channel has constant gain and linear phase response over range of frequencies which is less than the signal bandwidth then the channel creates frequency selective fading. That's different frequency components are faded differently. In time domain the received signal is a distorted because of multiple delayed and faded instances of transmitted signal. As signal gets dispersed in time domain, so channel induces ISI (Inter Symbol Interference). In terms of channel parameters

$$\text{If } f_m > f_0 \text{ and } T_s < \sigma_\tau$$

Then the channel creates frequency selective fading

In a fast fading channel, the channel characteristics changes multiple times within the symbol duration that's it changes at a rate higher than that of the transmitted signal. So this causes frequency dispersion which happens because of high Doppler spreading. We can say low data rate signals have more chance of being fast faded. Thus

The signal suffers fast fading if

$$T_s > T_c \text{ and } f_m < f_d$$

In a slow fading channel, the channel impulse response changes at a rate much lower than that of the transmitted signal. In time domain the channel characteristics remain almost constant during one symbol time. The Doppler spread here is less as compared bandwidth of the baseband signal. Thus

The signal suffers fast fading if

$$T_s < T_c \text{ and } f_m > f_d$$

So we can say the relative motion between mobile and receiver determines the channel to be slow fading or fast fading.

2.6 Rayleigh and Ricean Distribution

In a multipath channel if the propagation delays due to multi paths becomes random and the no of multi paths becomes very large, then central limit theorem applies. So the received signal envelope becomes Gaussian and can be modeled using various distribution functions.

▪ Rayleigh Distribution

When phase and quadrature component of received envelope are independent and Gaussian with zero mean then the pdf of amplitude assumes Rayleigh distribution.

There's no line of sight path between transmitter and receiver.

The power is exponentially distributed.

Mostly used as it represents a general case.

▪ Ricean Distribution

Due to deterministic dominant term at least one of in-phase and quadrature component of the received envelope has non-zero mean. So now the pdf of received envelope assumes Ricean distribution.

There's a dominant line of sight path between transmitter and receiver.

It applies to microcellular systems.

III. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)

3.1 Introduction:

As we move ahead for higher generation of mobile technology we always encounter the need of high speed communication. Various multicarrier multiplexing techniques have evolved to meet these demands, some of them being code division multiple access (CDMA) and orthogonal frequency division multiplexing (OFDM). OFDM utilizes orthogonal subcarriers to transmit information parallelly. In a conventional serial data transmission, the symbols are transmitted sequentially, with the frequency spectrum of each data symbol allowed to occupy the entire bandwidth. In OFDM, the data is divided among large number of closely spaced carriers (frequency division multiplexing). This is not a multiple access technique, since no common medium is to be shared. Here only small amount of data is carried by each carrier, reducing the ISI significantly. Many modulation schemes could be used to modulate the data at a low bit rate onto each carrier. Bandwidth occupied by the OFDM systems being greater than the correlation bandwidth of the fading

channel gives it an extra edge over serial communication. Dividing an entire channel into many narrow sub bands makes the frequency response become relatively flat in each individual sub band. Since each sub channel covers only a small fraction original bandwidth, equalization is quite simple (differential encoding may even make equalization unnecessary). Use of guard interval, system's reaction to delay spread can be reduced. OFDM can be finally said as a form of multicarrier modulation where its carrier spacing is carefully selected so that each subcarrier is orthogonal to the other subcarriers.

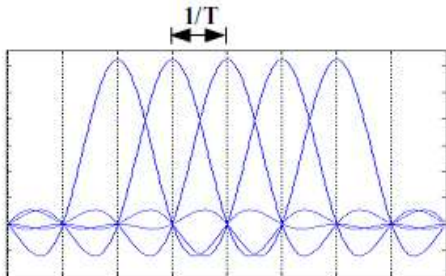


Fig. 3.1(a) OFDM transmission spectrum

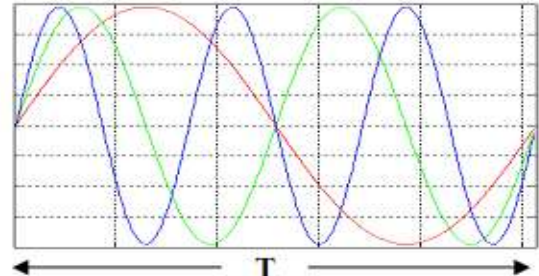


Fig. 3.1(b) waveform of carriers in OFDM

3.2 Importance of Orthogonality

The “orthogonal” part of OFDM name indicates there is some mathematical relationship between frequencies in sub bands. Introduction of guard bands reduces the spectral efficiency. So to enhance this efficiency, the carriers in OFDM signals are arranged in a manner such that individual carriers overlap and the signals can still be received without carrier interference.

Mathematically, signals are orthogonal if

$$\int_a^b \Psi_p(t) \Psi_q^*(t) dt = \begin{cases} K & \text{for } p = q \\ 0 & \text{for } p \neq q \end{cases} \quad (3.1)$$

Where * denotes the complex conjugate and interval [a b] is a symbol period.

An OFDM signal consists of a sum of subcarriers that are modulated by using BPSK, QPSK or QAM. Mathematically, each carrier can be described as a complex wave:

$$s_c(t) = A_c(t) e^{j[\omega_c t + \phi_c(t)]} \quad (3.2)$$

OFDM being carrying many carriers, its signal representation is:

$$s_s(t) = \frac{1}{N} \sum_{n=0}^{N-1} A_n(t) e^{j[\omega_n t + \phi_n(t)]} \quad (3.3)$$

Where

$$\omega_n = \omega_0 + n\Delta\omega$$

this is a continuous signal. If we consider the waveforms of each component of the signal over one symbol period, then $A_c(t)$ and $\phi_c(t)$ take on fixed values, which depends on the frequency of that particular carrier, and so can be rewritten as:

$$\phi_n(t) = \phi_n \quad \text{and} \quad A_n(t) = A_n$$

if now the signal is sampled at T time period, then the resulting signal becomes:

$$s_s(kT) = \frac{1}{N} \sum_{n=0}^{N-1} A_n e^{j[(\omega_0 + n\Delta\omega)kT + \phi_n]} \quad (3.4)$$

At this point, we restricted the time of analysis upto N samples. But its convenient to sample over one data symbol period. Thus we have:

$$\tau = NT$$

If we simplify eqn. 3.4, without the loss of generality by letting $\omega_0 = 0$, then the signal becomes:

$$s_s(kT) = \frac{1}{N} \sum_{n=0}^{N-1} A_n e^{j\phi_n} e^{j(n\Delta\omega)kT} \quad (3.5)$$

Which can now be compared with the general form of inverse fourier transform:

$$g(kT) = \frac{1}{N} \sum_{n=0}^{N-1} G\left(\frac{n}{NT}\right) e^{j2\pi nk/N} \quad (3.6)$$

Eqns 3.5 and 3.6 are equivalent if:

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{1}{NT} = \frac{1}{\tau} \quad (3.7)$$

This is the same condition that was required for orthogonality. Thus, maintaining orthogonality is that the OFDM signal can be defined by using Fourier transform procedures.

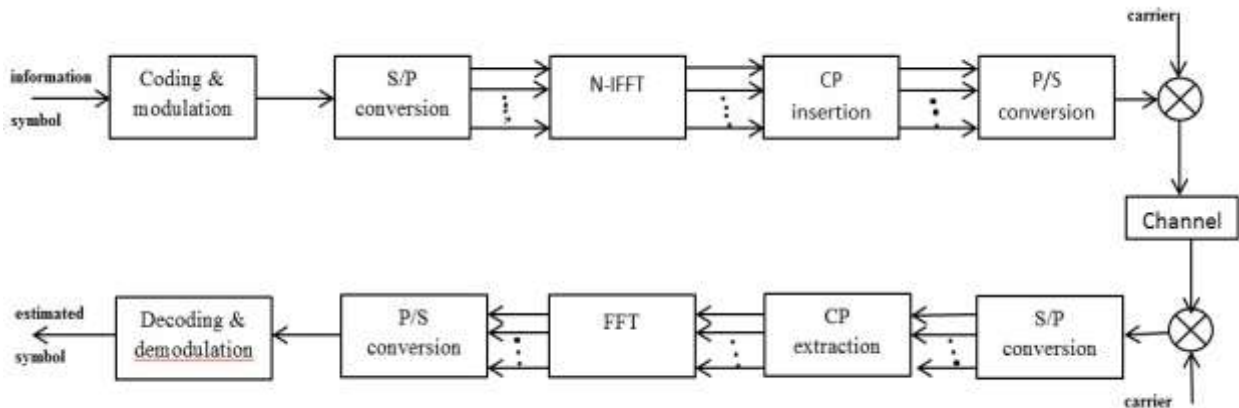


Fig. 3.2 Block Diagram of OFDM

3.3 Guard Interval

Individual sub channels can be completely separated by the FFT at the receiver when there are no ISI and ICI introduced by channel distortion. Practically these conditions cannot be obtained. Since the spectra of an OFDM signal is not strictly band limited, linear distortion such as multipath fading cause sub channel to spread energy in the adjacent channels. This problem can be solved by increasing symbol duration. One way to prevent ISI is to create a cyclically extended guard interval, where each symbol is preceded by a periodic extension of the signal itself. The total symbol duration being increased to $T_{total} = T_g + T$. When T_g is longer than the channel impulse response, the ISI can be eliminated. Since the insertion of guard interval will reduce data throughput, T_g is usually less than $T/4$.

The main reasons to use a cyclic prefix for the guard band interval are:

1. To maintain the receiver carrier synchronization.
2. Cyclic convolution can still be applied between the OFDM signal and the channel response to model the transmission systems.

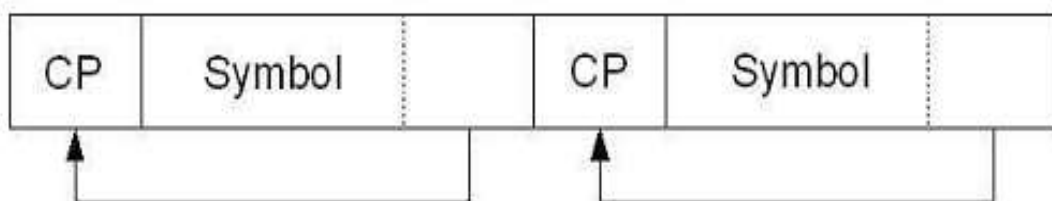


Fig. 3.3 Insertion of cyclic prefix

3.4 OFDMA

Like OFDM, OFDMA (Orthogonal frequency division multiple access) employs multiple closely spaced sub-carriers, but the subcarriers are divided into groups of subcarriers. Each group is named a sub channel. The sub-carriers that form a sub-channel need not be adjacent. In the downlink, a sub channel may be intended for different receivers. In the uplink, a transmitter may be assigned one or more sub-channels. Sub-channelization defines sub-channels that can be allocated to subscriber stations depending on the channel conditions and data requirements. Using subchannelization, within the same

time slot a mobile base station can allocate more transmit power to user devices with low SNR and vice-versa. This also save a user device transmit power as it can concentrate power only on certain sub-channels allocated to it.

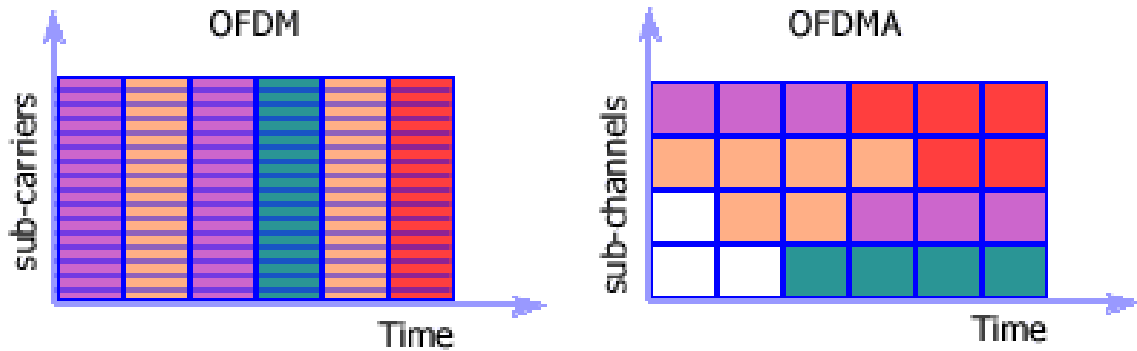


Fig. 3.4 Difference between OFDM and OFDMA

Apart from having certain advantages it could have from OFDM, the OFDMA waveform exhibits very pronounced envelop deviation resulting in a high PAPR (peak to average power ratio). And the signals having high PAPR requires highly linear power amplifiers like classA, class AB etc. to avoid excessive inter modulation distortion. To achieve this linearity, the amplifiers have to operate with a large back off from their peak power, resulting in decreased power efficiency. Another problem with OFDMA is, while up linking there is an introduction of frequency offset among the different terminals that transmit simultaneously, destroying the concept of orthogonality.

3.5 Switch to SC-FDMA

To overcome this problem, 3GPP is working on a modified form of OFDMA for uplink transmissions in LTE (long term evolution) of cellular systems. An alternative approach was sought known as Single Carrier Frequency Division Multiple Access (SC-FDMA). As in OFDMA, the transmitters in an SC-FDMA system use different orthogonal frequencies (subcarriers) to transmit information symbols. However, they transmit the subcarriers sequentially, rather than in parallel. This reduces envelope fluctuation relative to OFDMA. So SCFDMA has inherently low PAPR than OFDMA. But now it has the problem of ISI. It can be removed by adaptive channel equalization algorithms in the frequency domain. Time domain equalization is very complex because of long channel impulse response in time domain and large tap size of filters. But using Discrete Fourier Transform (DFT) in frequency domain it's much easier because DFT size doesn't increase linearly with channel response.

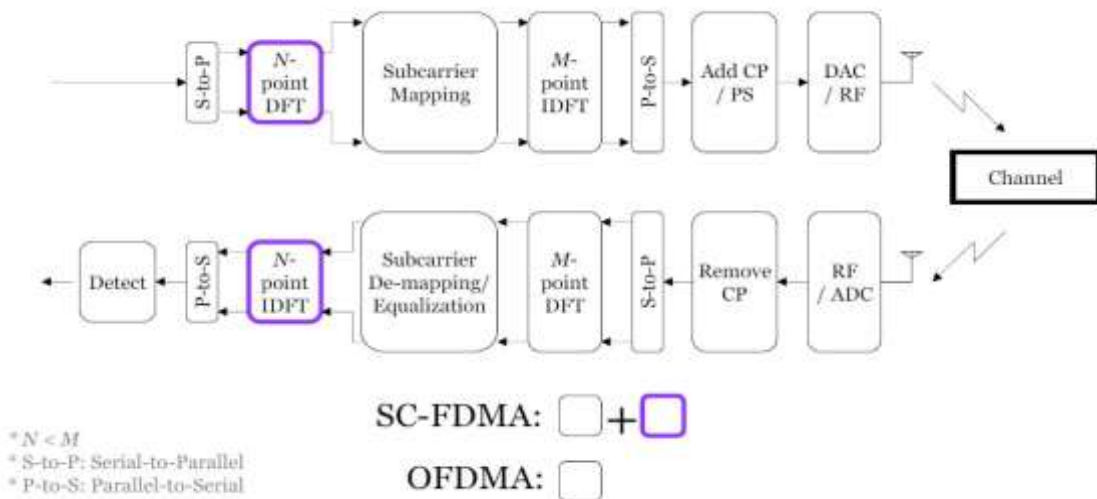


Fig. 3.5 Tx and Rx structure of SC-FDMA and OFDMA

3.6 Description of Problem Statement

As it's clear from the figure many blocks are common to both OFDMA and SCFDMA. At the input to the transmitter, a baseband modulator transforms the binary input to a multilevel sequence of complex numbers x_n in one of several possible modulation formats including Quaternary PSK (QPSK), 16-level quadrature amplitude modulation (16-QAM)

and 64-QAM etc. Then serial bit stream is converted to parallel bit stream of N data points. The first step is to produce a frequency representation X_k of the input symbols. It then maps each of the N DFT outputs to one of the M (>N) orthogonal subcarriers that can be transmitted, where $M=N*Q$, Q is the bandwidth expansion factor of symbol sequence.

The mapping can be of two types:

1. LFDMA
2. IFDMA

In **Localized FDMA** each terminal uses a set of adjacent subcarriers to transmit its symbols. Thus the bandwidth of an LFDMA transmission is confined to a fraction of the system bandwidth.

In **Interleaved FDMA** the subcarriers used by a terminal are spread over the entire signal band.

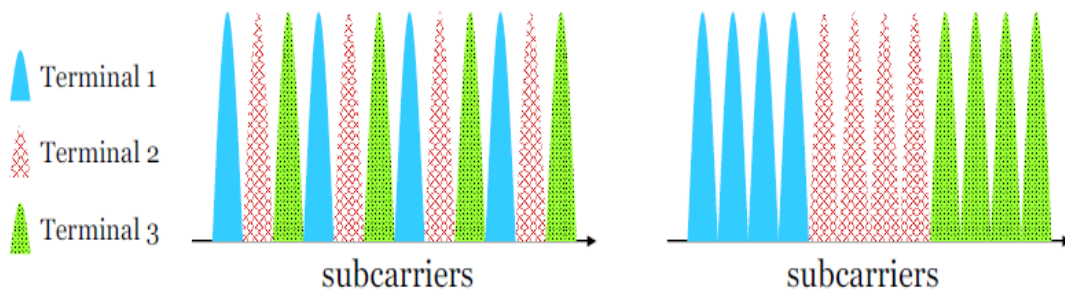


Fig. 3.6 Sub-carrier mapping for 3 users, 12 sub-carriers and 4 sub-carriers per user.

Fig 2 shows two type of mapping in the frequency domain. There are three terminals, each transmitting symbols on four subcarriers in a system with a total of 12 subcarriers. SCFDMA is better against frequency selective fading because its information is spread across the entire signal band. On the other hand, LFDMA can potentially achieve multi-user diversity in the presence of frequency selective fading if it assigns each user to subcarriers in a portion of the signal band where that user has favorable transmission characteristics.

After sub-carrier mapping we get the set of M complex sub-carrier amplitudes X_i frequency domain. Then M-DFT is performed to convert them into M time domain signals x_m . Each x_m then modulates a single frequency carrier and all the modulated symbols are transmitted sequentially.

3.7 Mathematical Calculation for PAPR

Let the data block of length N be represented by a vector $X=[X_0, X_1, \dots, X_{N-1}]^T$. Duration of any symbol X_k in the set X is T and represents one of the sub-carriers set. As the N sub-carriers chosen to transmit the signal are orthogonal, so we can have $f_n = n\Delta f$, where $n\Delta f = 1/NT$ and NT is the duration of the OFDM data block X. The complex data block for the OFDM signal to be transmitted is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi n\Delta f t}, \quad 0 \leq t \leq NT,$$

The PAPR of the transmitted signal is defined as

$$PAPR = \frac{\max_{0 \leq t < NT} |x(t)|^2}{1/NT \int_0^{NT} |x(t)|^2 dt}$$

The cumulative distribution function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold.

The CDF of the PAPR of the amplitude of a signal sample is given by

$$F(z) = 1 - \exp(-z)$$

The CCDF of the PAPR of the data block is desired in our case is to compare outputs of various reduction techniques. This is given by:

$$\begin{aligned} P(\text{PAPR} > z) &= 1 - P(\text{PAPR} \leq z) \\ &= 1 - F(z)^N \\ &= 1 - (1 - \exp(-z))^N \end{aligned}$$

3.8 Significance of pulse shaping filter in PAPR analysis

In digital communication, pulse shaping is one of the methods of changing the waveform of the transmitted pulse. It helps in limiting the effective bandwidth of the transmission and also the ISI caused by the channel can also be kept in control. Nyquist ISI criterion is the commonly used criterion for evaluation of filters. Examples of pulse-shaping filters are:

- Sinc filter
- Raised cosine filter
- Gaussian filter

We have implemented the raised cosine filter and root raised cosine filter in our work.

Raised cosine filter: Raised-cosine filter is practical to implement and it is in wide use. It has a parametrisable excess bandwidth, so communication systems can choose a trade-off between a more complex filter and spectral efficiency.

Its frequency-domain description is a piecewise function, given by

$$H(f) = \begin{cases} T, & |f| \leq \frac{1-\beta}{2T} \\ \frac{T}{2} \left[1 + \cos \left(\frac{\pi T}{\beta} \left[|f| - \frac{1-\beta}{2T} \right] \right) \right], & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$

β being the roll-off factor, is the measure of the excess bandwidth of the filter. As we increase the roll off factor the bandwidth gets increased but the PAPR gets reduced.

IV. CHANNEL ESTIMATION IN OFDMA

4.1 Introduction

A wideband radio channel characterizes frequency selective and also time variant. In both the frequency and time domain, the channel impulse response at different subcarriers, appear unequal. So we need to estimate the state of the channel at every instant. Pilot based approaches are widely implemented to estimate the channel characteristics and to correct the corrupt channel due to multipath fading. We have basically two kinds of pilot arrangement depending on the nature of channel. They are:

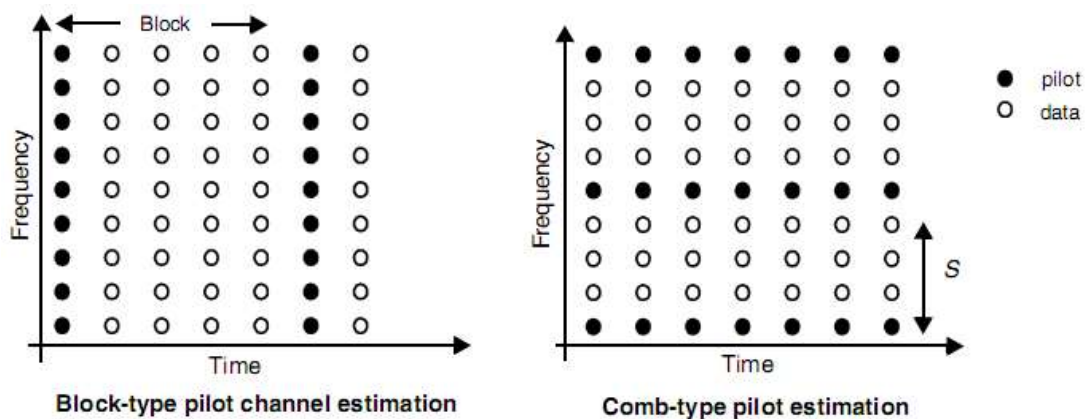


Fig. 4.1 two basic types of pilot arrangement for OFDM channel estimation

4.2 Block type of Pilot Arrangement

The first one, block-type pilot channel estimation, is developed under the assumption of slow fading channel, and is performed by inserting pilot tones into all subcarriers of OFDM symbols within a specific period. As the training block contains all the pilots, channel interpolation in frequency domain is not required. So this type of pilot can be said to be insensitive to frequency selectivity. As the coherence time is higher than the symbol period in slow fading due to lower doppler spread, the channel characteristics remains almost static for one symbol block time duration.

4.3 Comb Type of Pilot Arrangement

The second kind of pilot arrangement is denoted as comb-type pilot arrangement. Assuming the payloads of the pilot arrangements are the same, the comb type pilot arrangement has higher re-transmission rate. Thus the comb-type pilot arrangement gives better resistance to fast fading channels. Since only few subcarriers contain the pilot signal, the channel impulse response of non-pilot sub-carriers can be estimated by either linear, cubic or spline interpolation of the neighboring pilot sub-carriers. We can conclude that such pilot arrangement is sensitive to frequency selectivity. As the coherence time is less than the symbol period in fast fading due to higher spread, the channel characteristics fluctuates many time within one symbol block time duration.

4.4 Our Working Environment

As we are working on a Rayleigh slow fading channel we have stressed on the various block type pilot arrangement of channel estimation.

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, and all the subcarriers are used as pilots. So in our work we are using a general model for a slowly fading channel, where we make use of MMSE (minimum mean square error) and LS (least square) estimator and a method for modifications compromising between complexity and performance. The use of DPSK (differential phase shift keying) in OFDM systems avoids the tracking of a time varying channel. However, this limits the number of bits per symbol and results in 3 decibels loss in SNR. If we have a channel estimator in the receiver side, multi amplitude signaling schemes or M-ary PSK modulation schemes can be used. We have worked on BPSK, QPSK, 16 QAM modulation schemes for this purpose. Now we will look into the various estimation techniques in details and compare the biasedness, complexity and performance of each. Performance is presented both in terms of Mean Square Error (MSE) and Symbol Error Rate (SER).

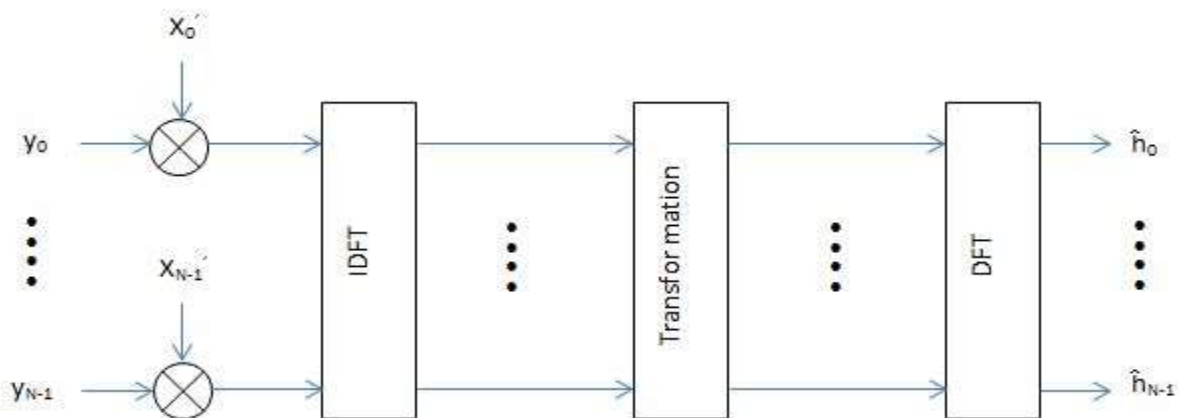


Fig. 4.2 general estimator structure

4.5 Mathematical analysis of the channel estimators

$$Y = XFg + W = Xh + W$$

Where:

$$X = \text{diag} \{ X_0, X_1, \dots, X_{N-1} \}$$

$$Y = [Y_0, Y_1, \dots, Y_{N-1}]^T$$

$$W = [W_0, W_1, \dots, W_{N-1}]^T$$

$$h = [h_0, h_1, \dots, h_{N-1}]^T = \text{DFT}_N \{ g \}$$

F = DFT transform block

4.5.1 Minimum Mean Square Error Estimation

MSE (mean square error) is expressed as:

$$J(e) = E[(h - \hat{h})^2] = E[(h - \hat{h})^H (h - \hat{h})]$$

Invoking the well-known orthogonality principle in order to minimize the mean square error vector $e = h - \hat{h}$ has to be set orthogonal by the MMSE equalizer to the estimators input vector Y

$$E[(h - \hat{h}) Y^H] = 0$$

$$\Rightarrow E[HY^H] - ME[YY^H] = 0$$

$$\Rightarrow E[FGY^H] - ME[YY^H] = 0$$

If the time domain channel vector g is Gaussian and uncorrelated with the channel noise W , then

$$FR_{Yg} = MR_{YY}$$

$$\text{Where } R_{Yg} = E[gY^H] \text{ and } R_{YY} = E[YY^H]$$

$$R_{Yg} = E[g(XFg + W)^H] = R_{gg} F^H X^H$$

As $E[gW^H] = 0$ i.e. g is uncorrelated with W

$$R_{YY} = E[YY^H] = E[(XFg + W)(XFg + W)^H] = XFR_{gg}F^H X^H + \sigma_n^2 I_N$$

Where σ_n^2 is the variance of the noise.

$$\text{And } M = FR_{Yg}R_{YY}^{-1}$$

$$\hat{h} = FR_{Yg}R_{YY}^{-1}Y$$

The time domain MMSE estimate of h is given by

$$\hat{g}_{MMSE} = R_{Yg}R_{YY}^{-1}Y$$

So Q block in the fig. 4.2 for MMSE estimator is given by

$$Q_{MMSE} = R_{gg}[(F^H X^H X F)^{-1} \sigma_n^2 + R_{gg}]^{-1} (F^H X^H X F)^{-1}$$

4.5.2 Least Square Error Estimation

We have to minimize

$$J = (Y - Xh)^H (Y - Xh)$$

$$\text{i.e. } \left. \frac{\partial J}{\partial H} \right|_{\hat{H}} = 0$$

$$\hat{h} = X^{-1}Y$$

Time domain LS estimate of g is given by

$$\hat{g} = F^H X^{-1}Y$$

So Q block in the fig. 4.2 for LS estimator is given by

$$Q_{LS} = (F^H X^H X F)^{-1}$$

The MMSE estimator suffers from higher complexity because it requires the calculation of an $N \times N$ matrix Q_{MMSE} , whose complexity increases with increase in N . and LS estimator gives higher mean square error. So we need to move on to another kind of estimator which would overcome the drawback of both the methods.

4.6 Modified Mmse Estimator

A straightforward way of decreasing the complexity is to reduce the size of Q_{MMSE} . As most of the energy in g is contained in, or near, the first L taps as shown in fig., so here we would study a modification of MMSE estimator, where only the taps with significant energy are considered. The components in R_{gg} corresponding to low energy taps in g are approximated to zero.

So R_{gg} is a $L \times L$ matrix containing the covariance of first L components of g . The DFT matrix also needs modification for finding DFT of such matrix. Now it would be an $N \times L$ matrix by taking only the first L columns of DFT matrix. If T denotes the modified DFT matrix then as shown in [3]

$$\hat{h}_{MMSE} = T Q_{MMSE}^{-1} T^H X^H Y$$

where $Q'_{MMSE} = R'_{gg} [(T^H X^H X T)^{-1} \sigma_n^2 + R'_{gg}]^{-1} (T^H X^H X T)^{-1}$

As L is a very fraction of N then the computational burden sharply decreases. As we know the LS estimator doesn't use the statistics of channel only depends on input and output. So modification to LS estimator isn't required as it won't relieve any computational burden.

V. SIMULATIONS AND RESULTS

5.1 Comparison of PAPR for OFDMA and SCFDMA

Parameters	Values
Data block size(N)	16
Q:M/N	32
Transmission bandwidth	5 Mhz
Over sampling factor	4
Number of runs	10

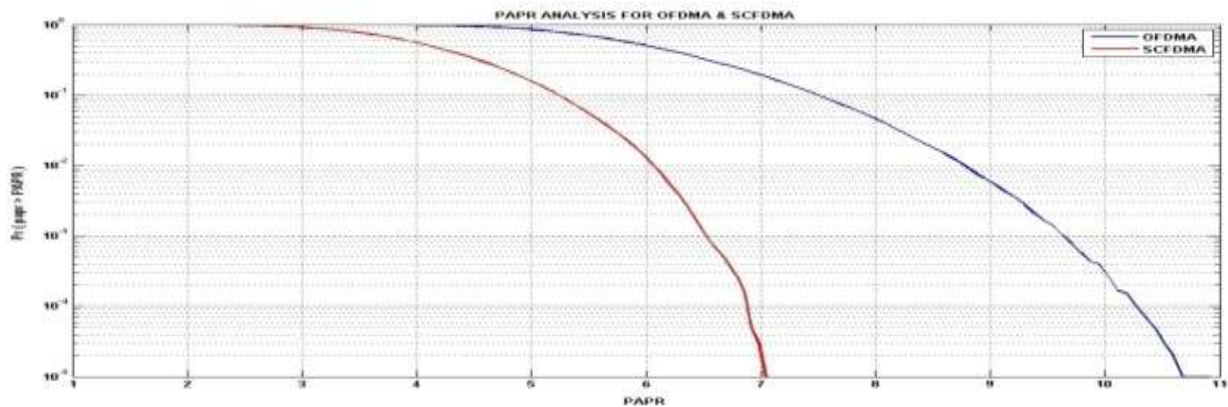


Fig. 5.1 CCDF of PAPR for OFDMA and SCFDMA

5.2 Comparison of PAPR for various kinds of SCFDMA techniques

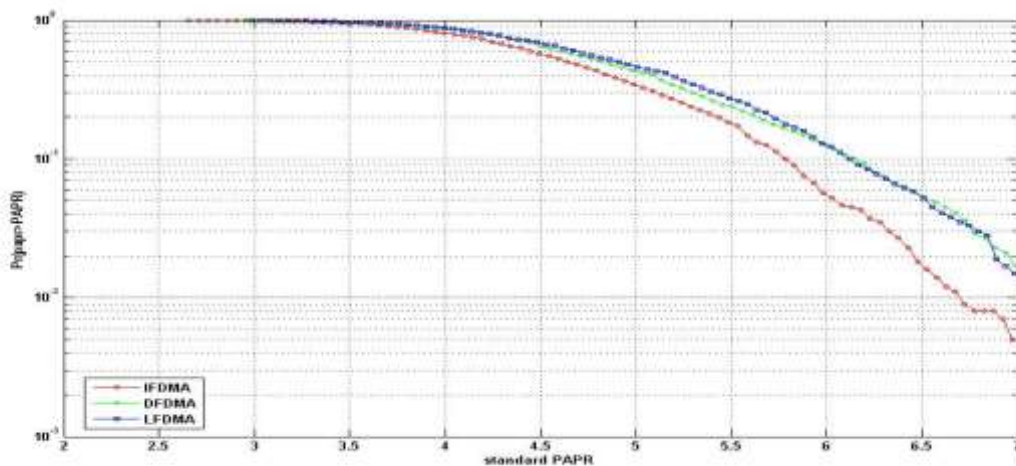


Fig. 5.2 CCDF of PAPR for various kinds of SCFDMA using 16QAM modulation

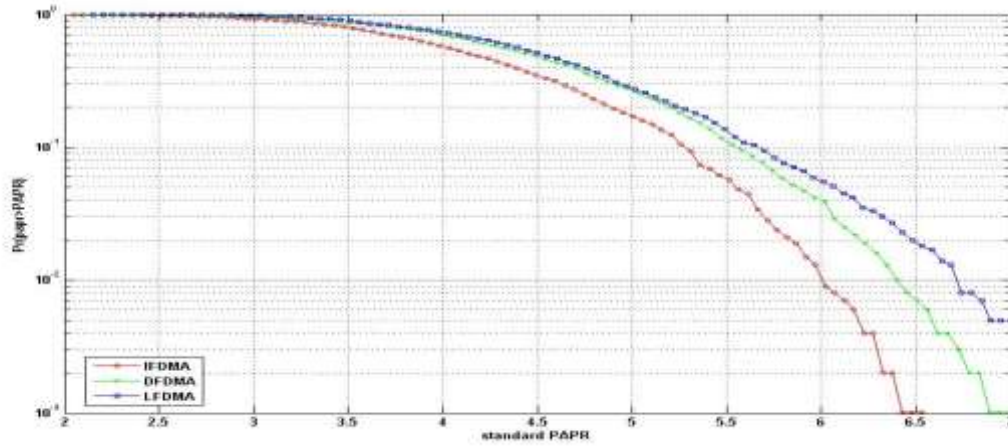


Fig. 5.3 CCDF of PAPR for various kinds of SCFDMA using QPSK modulation

Here we conclude that the PAPR for SCFDMA is found to be less than OFDMA. And among SCFDMA localized FDMA have higher PAPR compared to interleaved and distributed FDMA(As we can see from both the fig 5.2 and 5.3 for 16qam and qpsk modulation respectively)

5.3 Analysis of PAPR by altering Roll-off Factor of pulse shaping filter

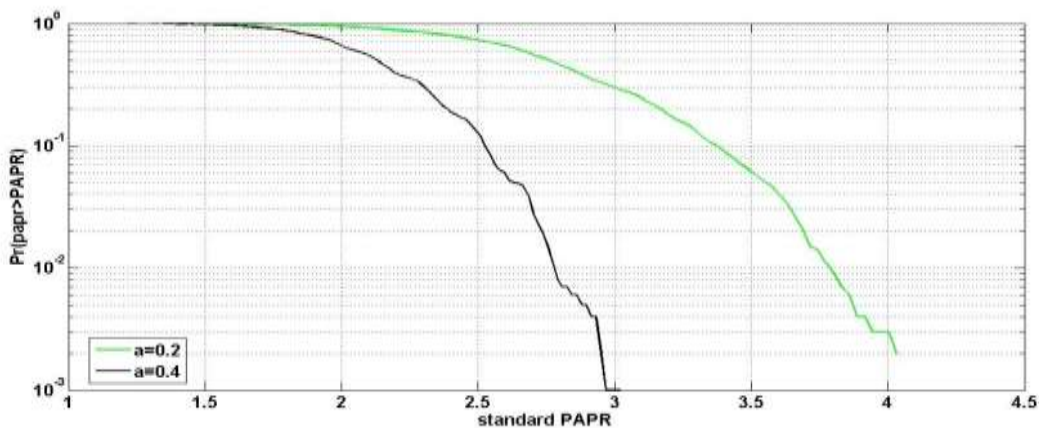


Fig. 5.4 CCDF of PAPR by altering the roll off factor of the pulse shaping filter used for lowering ISI

Here we can conclude that as we increase the roll off factor the PAPR reduces and also the impulse response decays much faster at the zero crossing. The bandwidth of the filter increases and the time side lobe levels decreases in adjacent symbol slots which is the basic need of ISI reduction. So this is the advantage we get by using a pulse shaping raised cosine filter with high roll off factor.

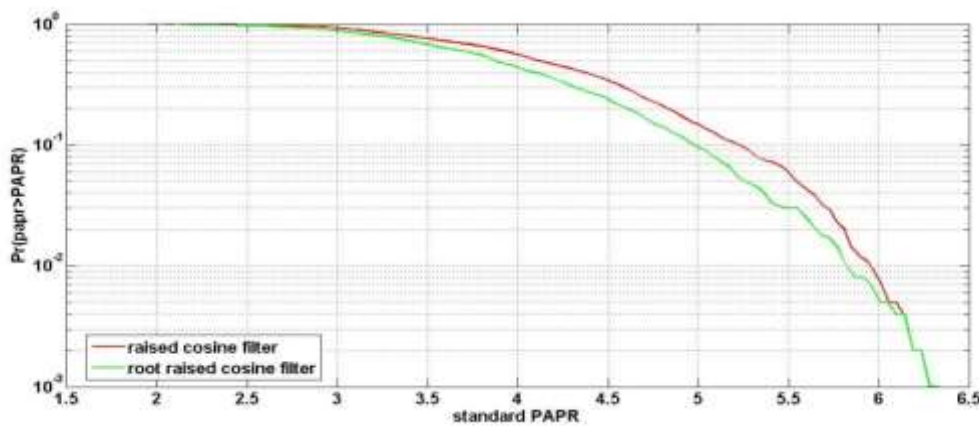


Fig. 5.5 CCDF of PAPR by using two different kinds of pulse shaping filter

Here we see that root raised cosine filter gives better PAPR response than that of the raised cosine filter

5.4 Channel Estimation

5.4.1 Simulation By Altering The Pilot Symbols Quantity

Using block type pilot arrangement channel estimation

Total number of sub channels: 256

Guard band interval: 64

Number of iteration: 500

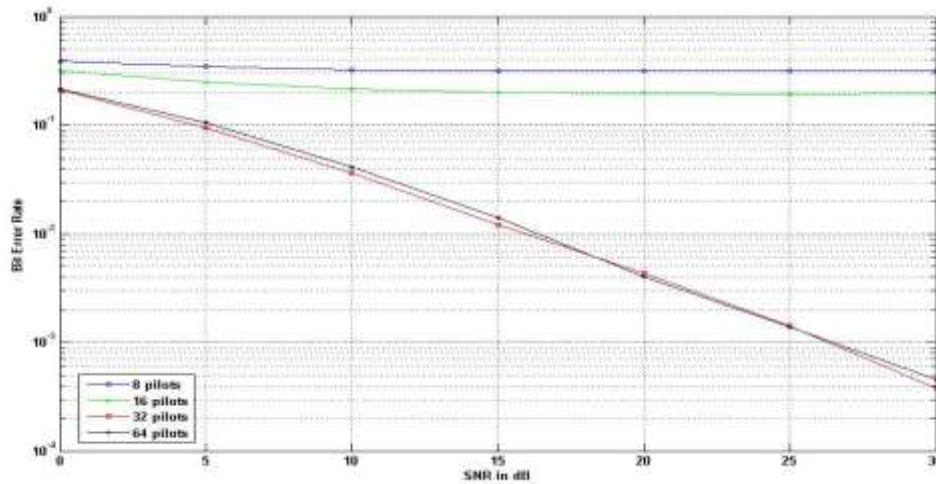


Fig. 5.6 BER v/s SNR plot for different numbers of pilot symbols used for channel estimation

Here we conclude that higher the number of pilot symbols is used more perfectly the channel is estimated and less is the bit error rate. But here the tradeoff is done with the amount of data sent within that time. As we increase the number of pilot symbols amount of data sent in that fixed time reduces.

5.4.2 Study and Plot of the channel characteristics

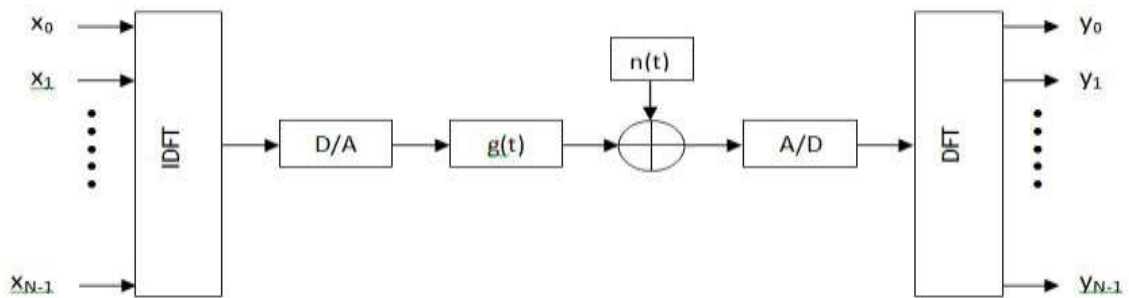


Fig. 5.7 baseband OFDM system

In matrix notation output y_s can be written as

$$Y = Xh + n$$

Y : output matrix

X : input diagonal matrix

h : sampled version of frequency response of $g(t)$

n : additive white gaussian noise

The channel used in our case is a Rayleigh slow fading channel which has two fractionally separated taps given by:

$$g(t) = \delta(t - 0.5T_s) + \delta(t - 3.5T_s)$$

T_s : sampling time of the system

g_k is the IDFT of sampled fourier transform of the above channel impulse response given by

$$g_k = \frac{1}{\sqrt{N}} \sum_m \alpha_m e^{-j\frac{\pi}{N}(k+(N-1)\tau_m)} \frac{\sin(\pi\tau_m)}{\sin(\frac{\pi}{N}(\tau_m - k))}$$

Simulation parameters:

No of OFDM symbols: 64

For mod-MMSE

No of taps from start: 10

No of taps from start and end: 15 + 15=30

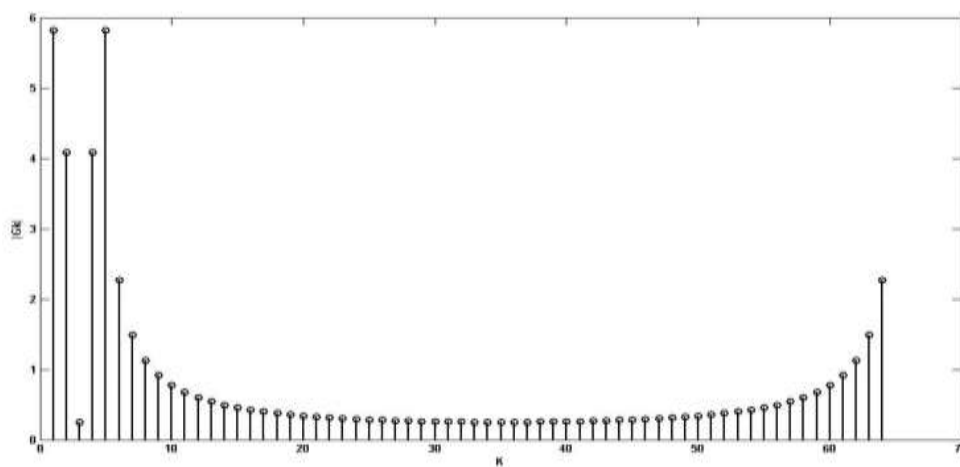


Fig 5.8 plot of $|g_k|$ v/s k

As it's clear from the above figure if τ_m would be an integer then all the energy from the tap α_m would map to g at τ_m . But as taps are fractionally spaced energy leaks to nearby taps as shown. After $k > 10$ there's 93% decrease from the peak value. So rest taps can be set to zero as they won't contribute significantly to the autocovariance matrix R_{gg} , which is the basis for mod-MMSE.

5.4.3 MSE and SER Plots For Various Estimation Techniques

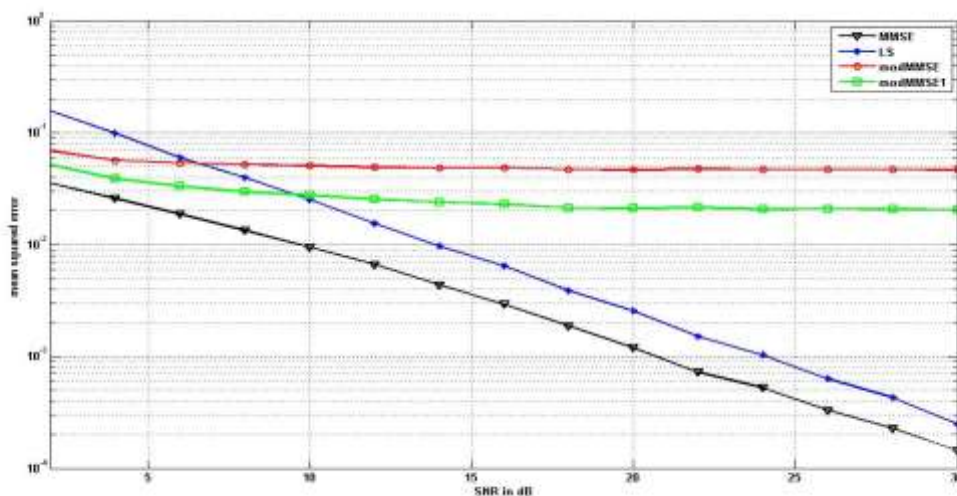


Fig. 5.9 MSE v/s SNR plot for LS, MMSE, modified MMSE(with 10 and 15 taps)estimator

So here we conclude that the MMSE estimator giving least mean square error, have better performance but due to its complexity we adopt modified MMSE. Here the performance is reduced but complexity is avoided. But as we increased the number of taps for consideration from g_k the error reduces.

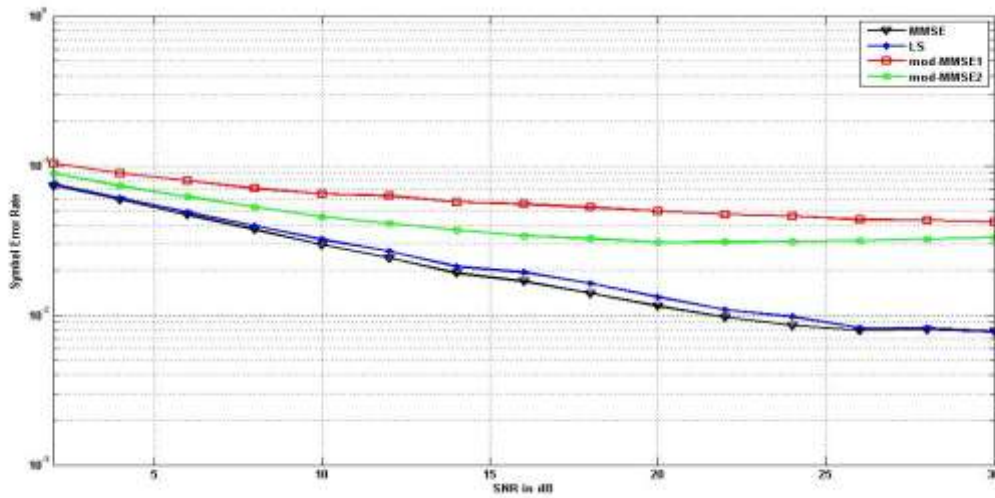


Fig. 5.10 SER v/s SNR plot for LS, MMSE, modified MMSE(with 10 and 15 taps)estimator

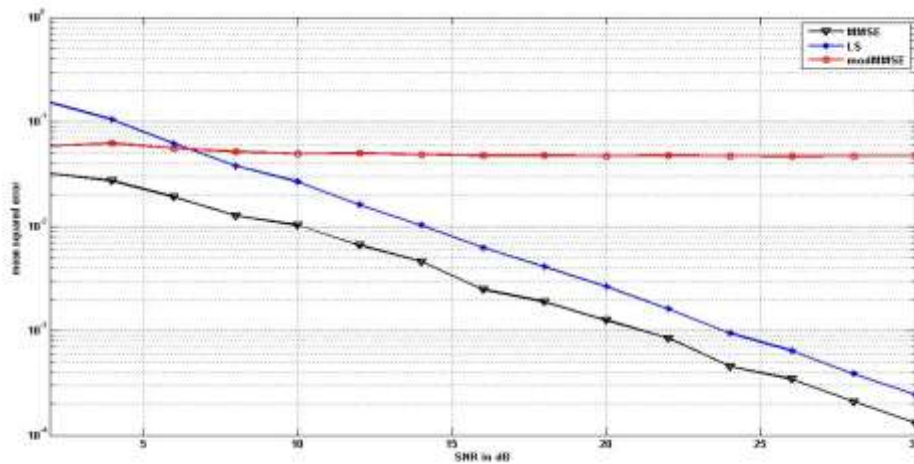


Fig. 5.11 MSE v/s SNR plot for LS,MMSE,modified MMSE estimator(using QPSK modulation)

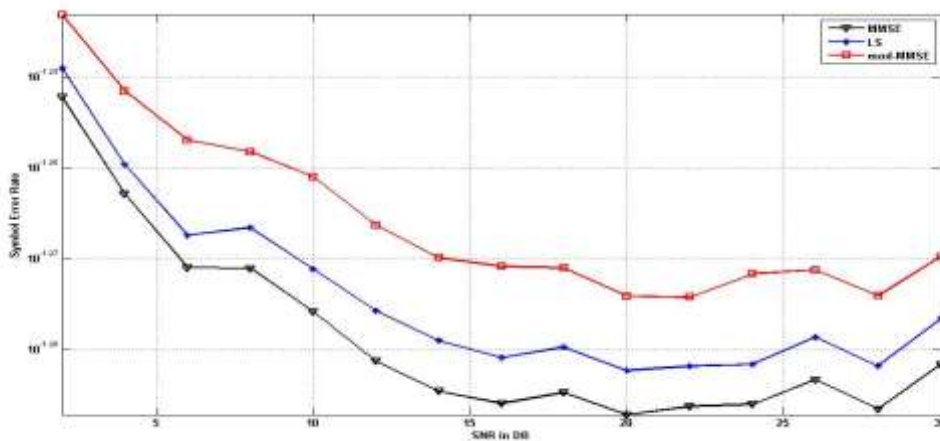


Fig. 5.12 SER v/s SNR plot for LS, MMSE, modified MMSE estimator (for QPSK modulation)

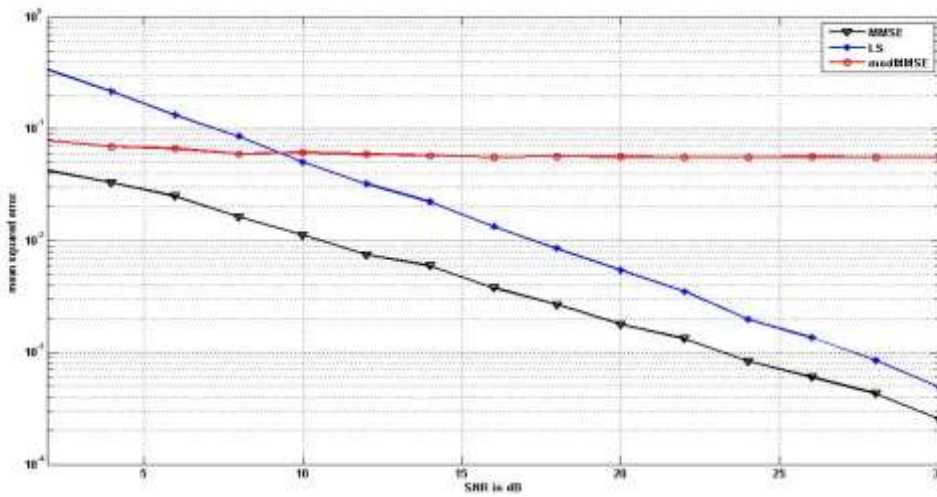


Fig. 5.13 MSE v/s SNR plot for LS, MMSE, modified MMSE estimator (using 16 QAM modulation)

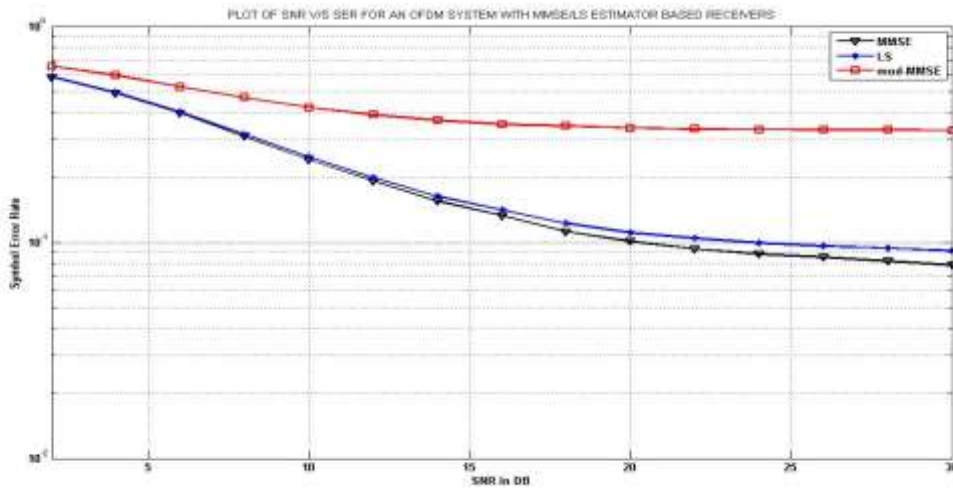


Fig. 5.14 SER v/s SNR plot for LS, MMSE, modified MMSE estimator (for 16 QAM modulation)

As can be seen from above plots that same nature of performance of estimators is preserved in different modulation techniques.

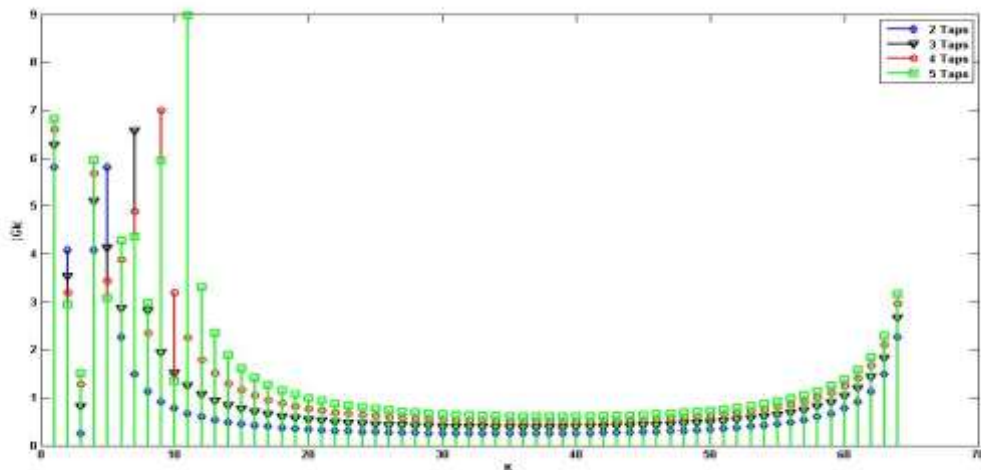


Fig 5.15 plot of g_k v/s k for different amount of multipath delays in $g(t)$.

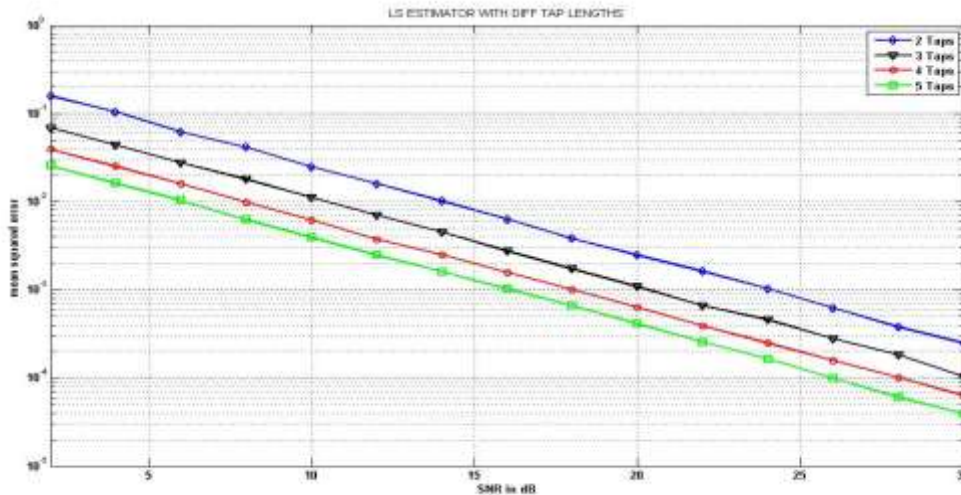


Fig. 5.16 MSE v/s SNR using LS estimator for different amount delay components in the channel

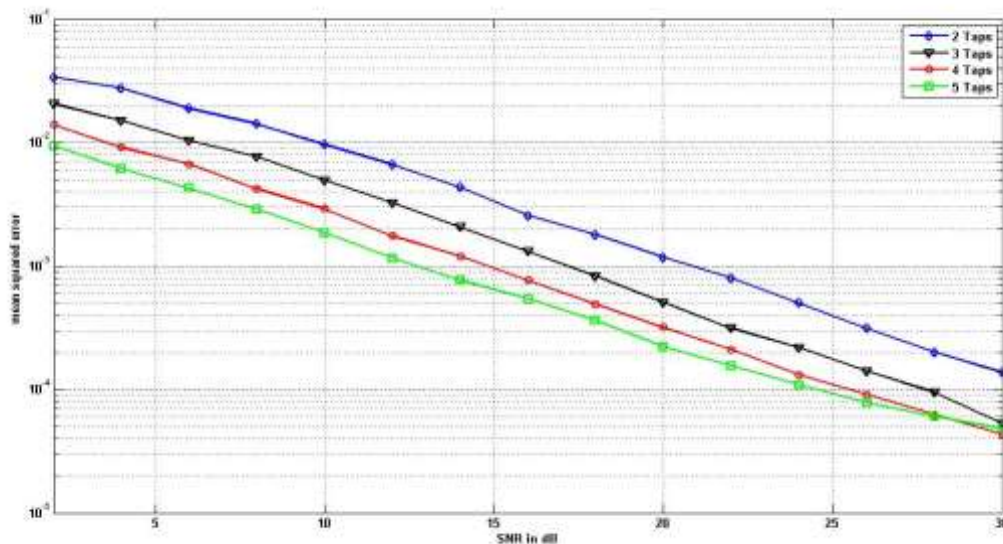


Fig. 5.17 MSE v/s SNR using MMSE estimator for different amount delay components in the channel

Here we conclude that as the number of delay components in the channel is increased the mean square error is reduced as more delay information is taken into consideration for channel estimation.

For different tap lengths we can't simulate using a fixed mod-MMSE scheme, as can be seen from Fig 5.16 the profile of $|g_k|$ changes with changing no of taps.

VI. CONCLUSION

PAPR analysis of OFDMA and SCFDMA showed that the latter shows less PAPR value so can be a better candidate for uplink in 3GPP LTE as accepted. Among the various kind of mapping of SCFDMA interleaved mapping gives less PAPR but has lesser throughput as compared to localized technique. But a tradeoff is used in the form of distributed mapping. By using pulse shaping filter with high roll off factor significant reduction of PAPR occurs but price paid is increased bandwidth requirement. So a middle value of roll off factor is chosen according to application.

By comparing the performance of different estimation techniques in terms of MSE and SNR we can say modified MMSE provides significant reduction in computation complexity but moderate MSE values. Increasing the SNR value beyond certain point will show that the LS gives best result with reduced complexity. But for moderate values of SNR modified MMSE gives satisfied result.

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