

STUDIES ON MIXED MODE FRACTURE: PREDICTION AND VERIFICATION

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Abstract: A novel mixed mode fracture test specimen - a semi-circular disc with an edge crack under three point bending is the focus of this investigation. finite element modelling using a commercial FEA software(ANSYS) is presented to compute accurate mixed mode stress intensity factors(K_I , K_{II}) as a function of crack lengths (a/R) and support locations ($S_2 = aR$). It is shown that for $0 < \alpha \leq 1$ full mode mixities from pure mode I to pure mode II could be achieved. Choosing the Strain Energy Density Theory of fracture and the material properties of PMMA Poly(methyl meth acrylate), predicted crack growth direction and fracture load are graphically presented. These are co-related with available mixed mode fracture test results. Comparisons are also made with candidate fracture criteria

Keywords: Mixed mode fracture, Strain Energy Density theory, PMMA, Stress Intensity Factor.

I. INTRODUCTION

The presence of flaws (cracks) are quite often inevitable in engineering structures and components. The cracks can be induced during the manufacturing processes, cyclic loading, or environmental causes, etc. fracture is defined as a mode of failure due to unstable propagation of a crack with applied stress. Fracture mechanics provides a methodology to study fracture in materials, components and structures. Fracture mechanics analysis is the basis for damage tolerant design methodology. The main objectives of fracture mechanics analysis are; 1) Derivation of crack tip stress field equations and definition of fracture mechanics parameters (Stress Intensity Factor K, Energy Release Rate G, Non Linear Energy Release Rate J, Crack Tip Opening Displacement CTOD); 2) Determination of K, G, J and CTOD for cracked body problems; 3) Prediction of residual strength as a function of crack size and ; 4) Fatigue crack growth life Prediction under constant amplitude, variable amplitude and spectrum loads.

The objective of this paper is to present a novel mixed mode fracture test specimen- a semi-circular disc with an edge crack under three point bending. Stress intensity factor solutions use of strain energy density theory of fracture and co-relation of predictions with fracture test results on PMMA are presented in the following sections.

Several Analytical, numerical and experimental methods have been suggested by researchers for determination of mixed mode stress intensity factors. Finite element modelling for computational fracture mechanics using ANSYS software is used to provide accurate stress intensity factors (K_I , K_{II}) for the problem on hand

Mixed mode fracture test results obtained using a semi-circular disc with an edge crack under three point bending made of PMMA are reported in [1]. These results are used to verify the predictions using the strain energy density theory of fracture.

The typical finite element model is presented in fig 2 and fig 3. A refined mesh of singular isoperimetric triangular element with 6 nodes (STRIA 6) with $NS = 72$, $\Delta a = a/100$ and a compatible mesh of quadrilateral elements quadratic in order with 8 nodes (QUAD8) in the rest of the domain is used.

II. FINITE ELEMENT MODELING

Fig 1 shows a semicircular disc with an edge crack subjected to three point bending. The dimensions of the specimen are $R = 60$ mm, $S1 = 40$ mm, $S2$ (variable from 6.07 mm to 40 mm), crack length $a = 20$ mm and thickness $t = 6$ mm. The typical finite element model is presented in Fig 2 and Fig 3. A refined mesh of singular isoperimetric triangular element with 6 nodes (STRIA 6) with $NS = 72$, $\Delta a = a/100$ and a compatible mesh of quadrilateral elements quadratic in order with 8 nodes (QUAD8) in the rest of the domain is used. Nodes at the support points are constrained in the Y direction and a point load of 1000 N is applied at the top.

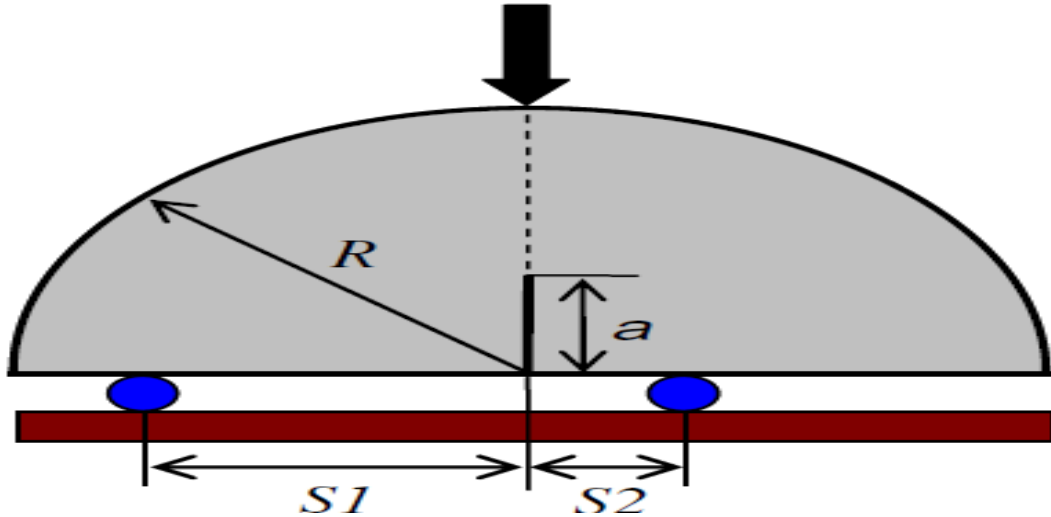


Fig 1: A test specimen to study mixed mode fracture

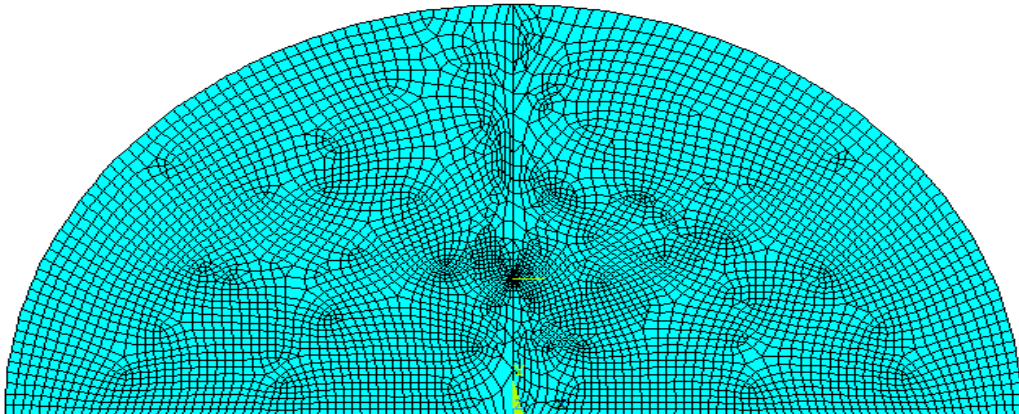


Fig 2: Mesh of SCB Specimen

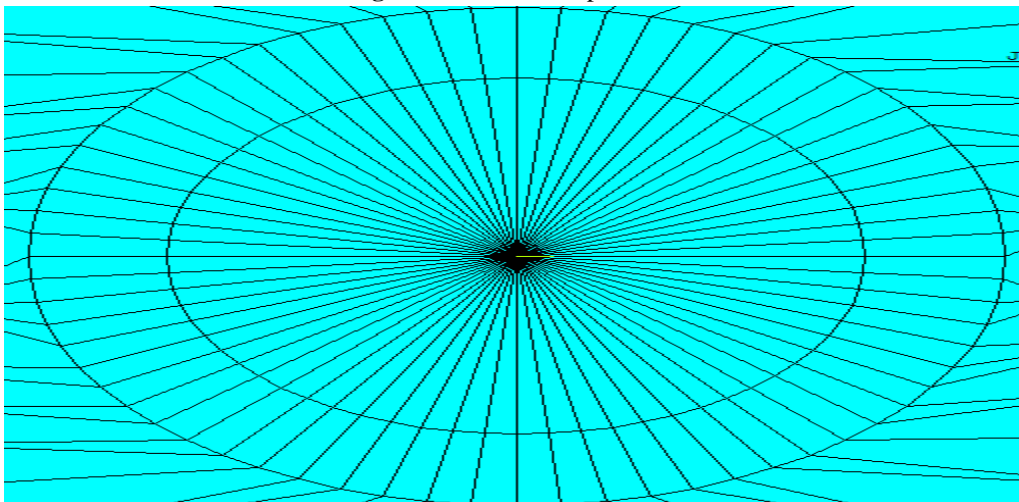


Fig 3: Crack tip singularity elements

III. STRESS INTENSITY FACTOR SOLUTIONS

The post processing command KCALC is used to calculate the stress intensity factors for different values of S2. The geometric factors Y_I and Y_{II} are calculated from the stress intensity factors K_I and K_{II} using the formulas.

$$K_I = \frac{P}{2Rt} \sqrt{\pi a} Y_I(a/R, S1/R, S2/R) \quad (1)$$

$$K_{II} = \frac{P}{2Rt} \sqrt{\pi a} Y_{II}(a/R, S1/R, S2/R) \quad (2)$$

The variation of geometric factors with S2 is presented in the Fig 4 and Fig 5.

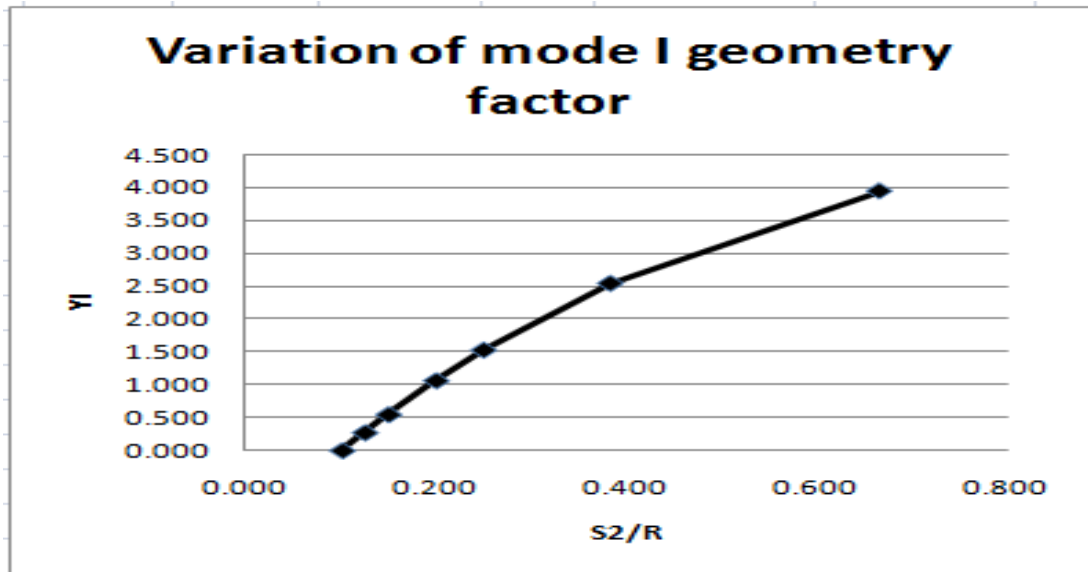


Fig 4: Variation of mode I geometry factor

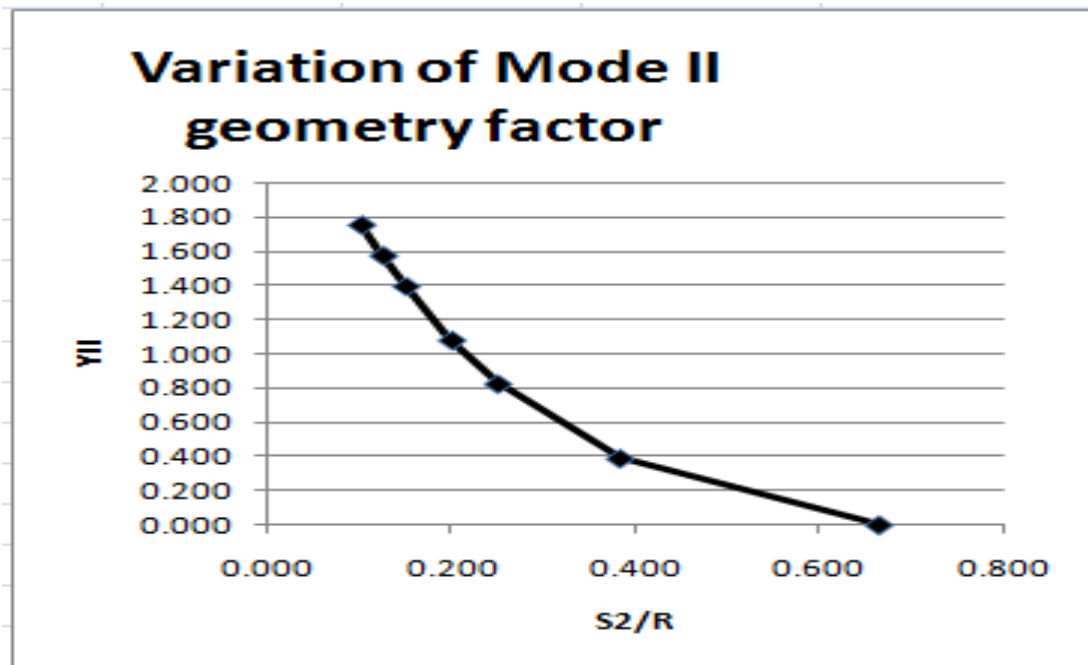


Fig 5: Variation of mode II geometry factor

Stress intensity factor solution for the problem onhand obtained using abaqus software are also reported in [1]. It is gratifying to note a close agreement between the two.

The graphical post processing capabilities in ANSYS is demonstrated by capturing the von Mises equivalent stress contours at the crack tip as shown in Fig 6 to Fig 9

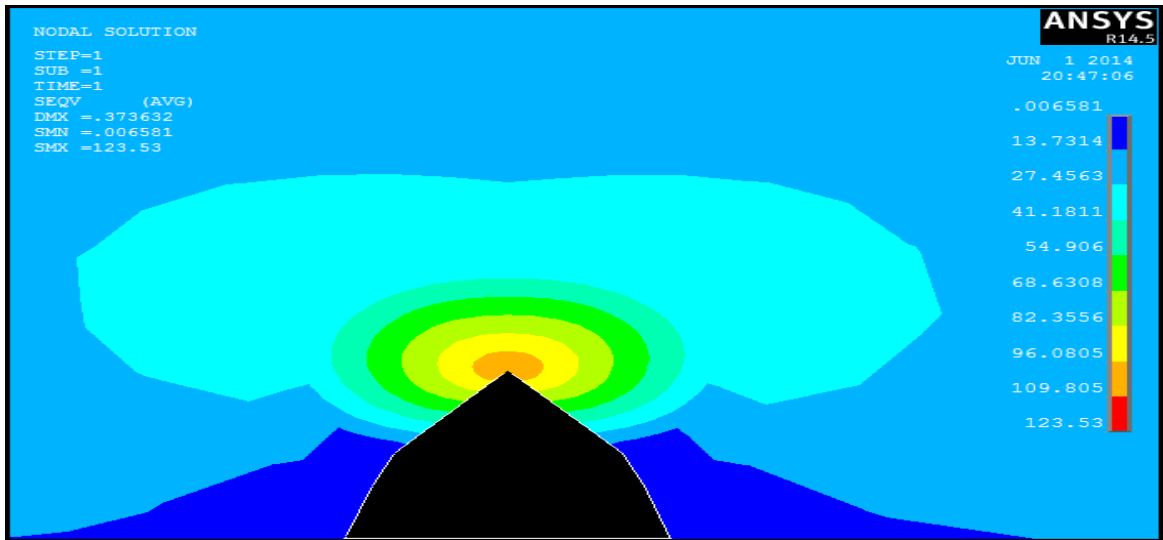


Fig 6: Von Mises stress plot at crack tip for mode I

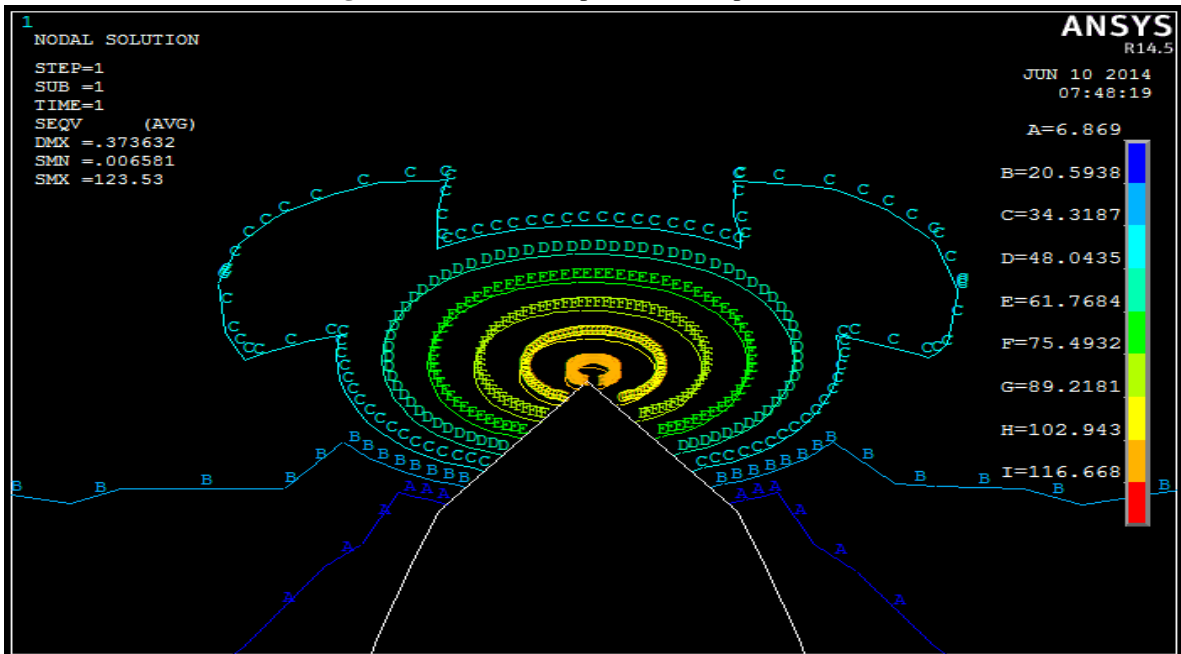


Fig 7: Von Mises stress line contour plot at crack tip for mode I

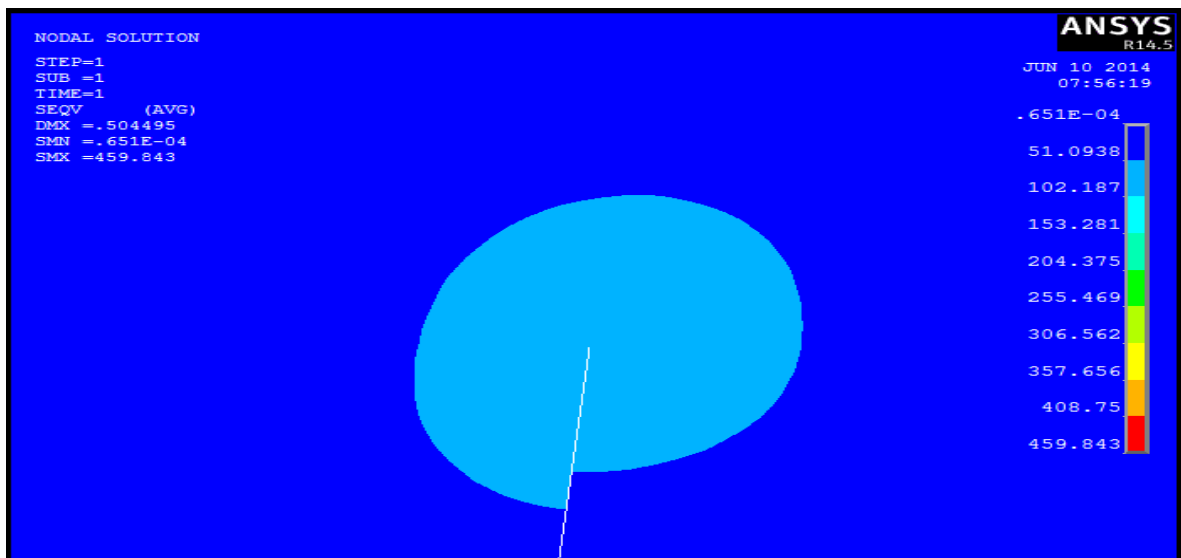


Fig 8: Von Mises stress plot at crack tip for mode I

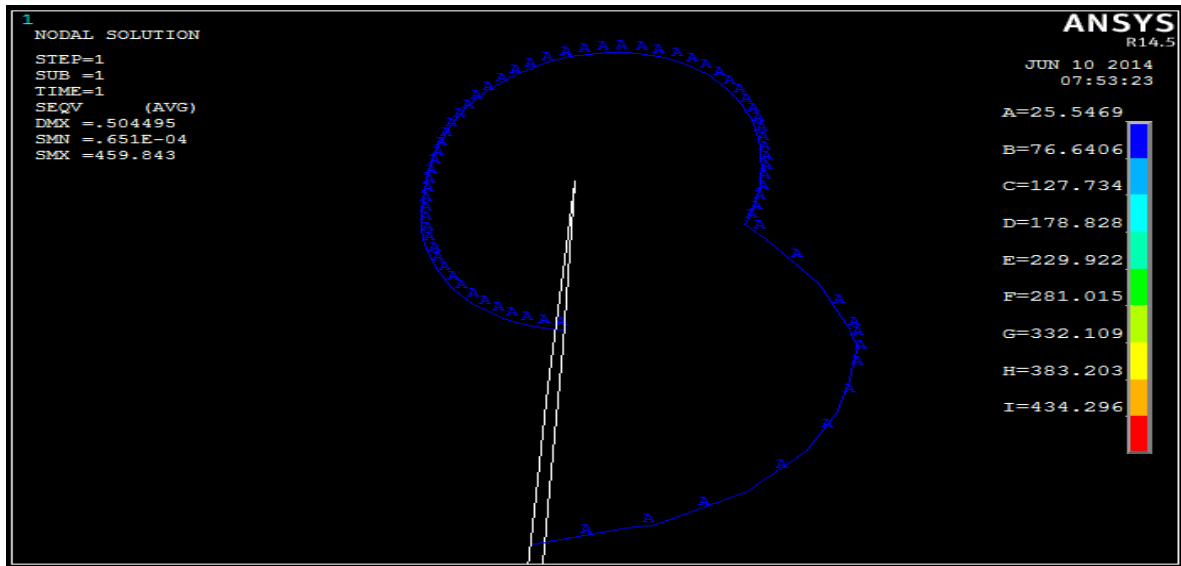


Fig 9: Von Mises stress line contour plot at crack tip for mode II

IV. STRAIN ENERGY DENSITY THEORY OF FRACTURE.

According to SED criterion, crack extension will occur in the direction of minimum strain energy density factor (S), and the extension will occur when the minimum value of S reaches a critical value S_c which is a material dependent parameter where

(3)

$$S(\theta) = [g_{11} K_I^2 + 2g_{12} K_I K_{II} + g_{22} K_{II}^2] / \pi$$

$$g_{11} = \frac{1}{16\mu} [(1 + \cos\theta)(\kappa - \cos\theta)]$$

$$g_{12} = \frac{1}{16\mu} \sin\theta [2\cos\theta - (\kappa - 1)]$$

$$g_{22} = \frac{1}{16\mu} [(\kappa + 1)(1 - \cos\theta) + (1 + \cos\theta)(3\cos\theta - 1)]$$

$$\mu = \frac{E}{2(1 + \nu)}$$

The conditions for predicting the direction of crack growth are

$$\frac{\partial S}{\partial \theta} = 0 \quad \frac{\partial^2 S}{\partial \theta^2} > 0 \tag{4}$$

The equation to calculate the crack growth direction θ is given as

$$[2\cos\theta - (k - 1)\sin\theta]K_I^2 + 2[2\cos 2\theta - (k - 1)\cos\theta]K_I K_{II} + [(k - 1 - 6\cos\theta)\sin\theta]K_{II}^2 = 0 \tag{5}$$

under the inequality condition

$$[2\cos 2\theta - (k - 1)\cos\theta]K_I^2 + 2[2(k - 1)\sin\theta - 4\sin 2\theta]K_I K_{II} + [(k - 1)\cos\theta - 6\cos 2\theta]K_{II}^2 > 0 \tag{6}$$

Crack extension will occur when the minimum value of strain energy density factor (S_{min}) reaches a critical value of strain energy density factor S_c thus the condition is given as

$$(S_{min}) \geq S_c \tag{7}$$

The material dependent critical value S_c is obtained for pure mode I condition with $\vartheta = 0$ and $K_I = K_{IC}$ as

$$S_c = \frac{K_{IC}^2}{4\pi E} (1 + \vartheta)(k - 1) \tag{8}$$

Thus, Inequality (7) is expressed as

$$S_{min} \geq \frac{K_{IC}^2}{4\pi E} (1 + \vartheta)(k - 1) \tag{9}$$

The predicted fracture surface for possible combinations of stress intensity factors KI and KII in the edge cracked semi circular disc in three point bending is presented in the Fig 10.

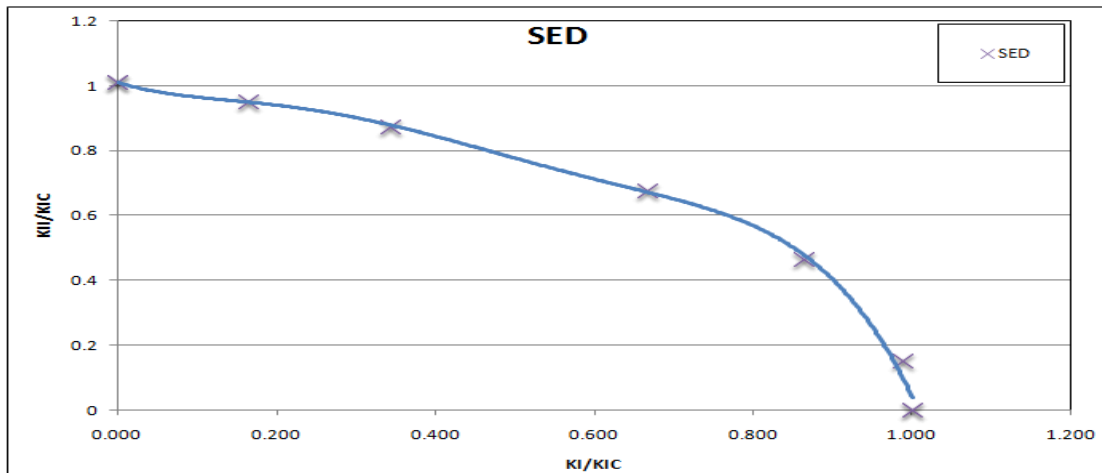


Fig 10: Predicted fracture surface curve using SED criterion

The predicted crack growth direction for possible combinations of stress intensity factors KI and KII in the edge cracked semi circular disc in three point bending is presented in the Fig 11.

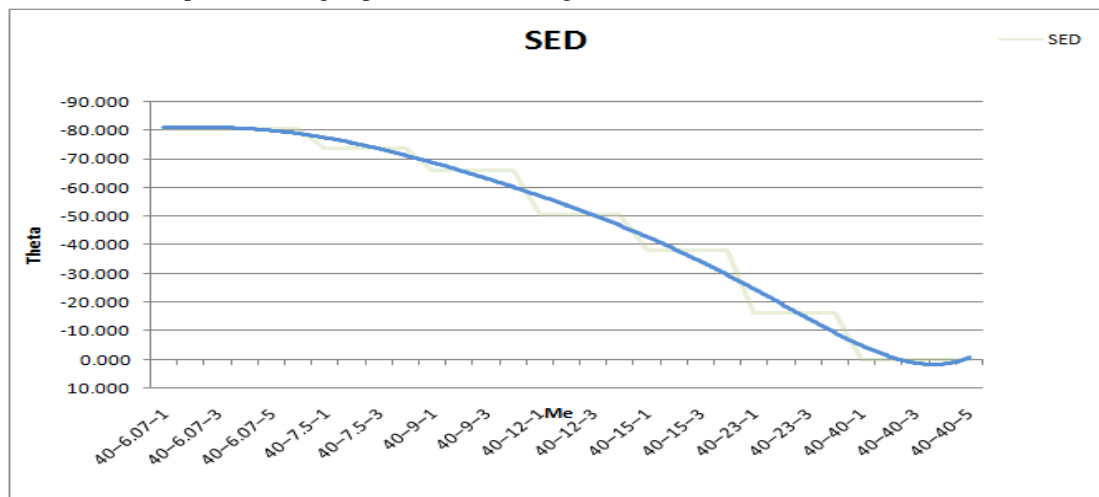


Fig 11: Predicted crack growth direction using SED Criterion

A graphical comparison of fracture surface and the crack growth direction with the experimental results of edge cracked semi circular disc in three point bending made out of PMMA reported in [1] is presented in the Fig 12 and Fig 13.

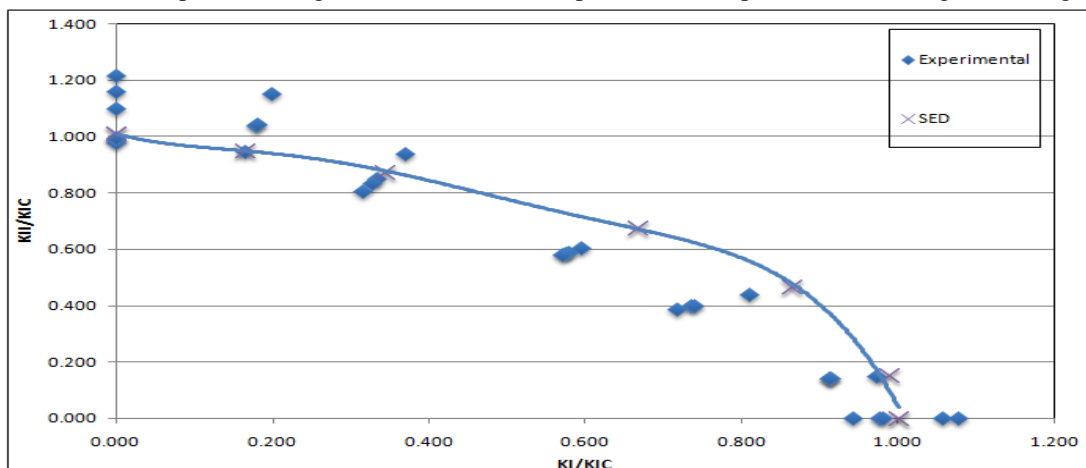


Fig 12: Comparison of experimental fracture load and those predicted using SED criterion

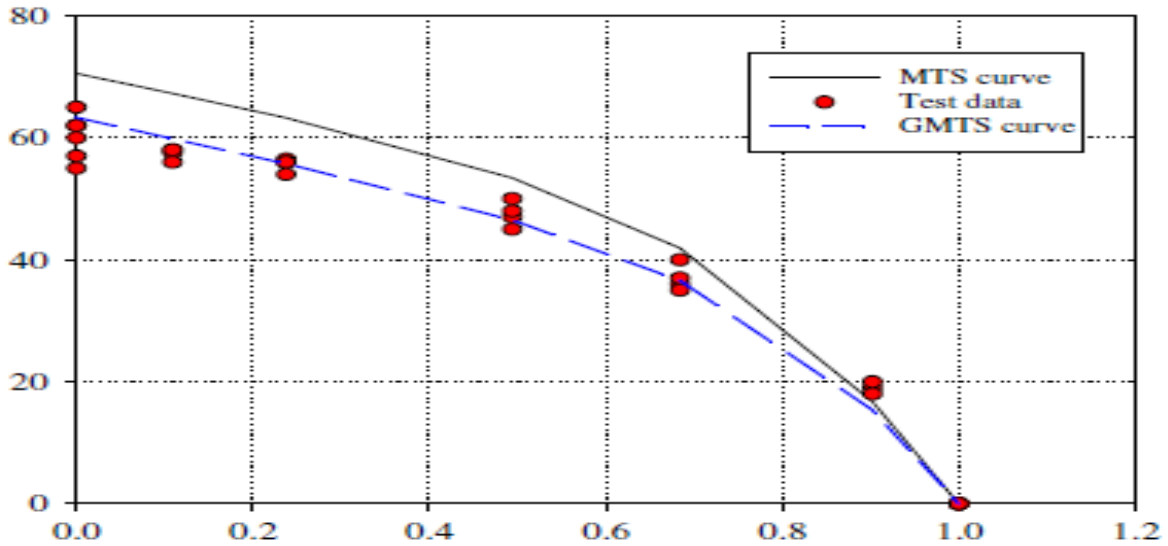


Fig 13: Figure showing the experimental crack growth direction [1]

A number of mixed mode fracture criteria in addition to SED have been proposed. Comparison of mixed mode fracture test results, Maximum Tangential Stress criterion(MTS), Generalised Maximum Tangential Stress criterion (GMTS), and SED criterion is presented in the Fig 14 and Fig 15.

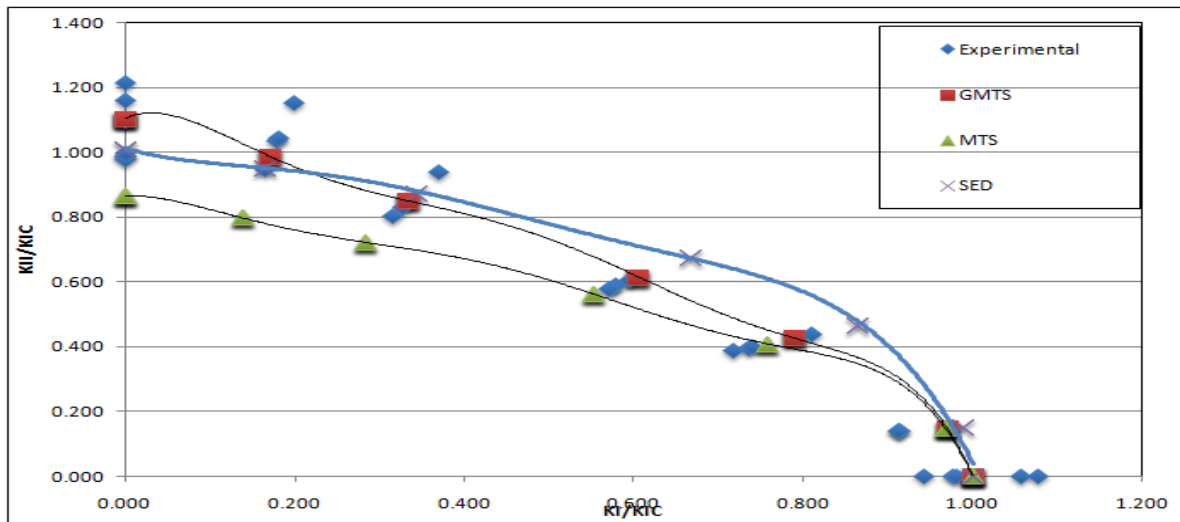


Fig 14: Comparison of experimental fracture load and those predicted using MTS,GMTS and SED criterion

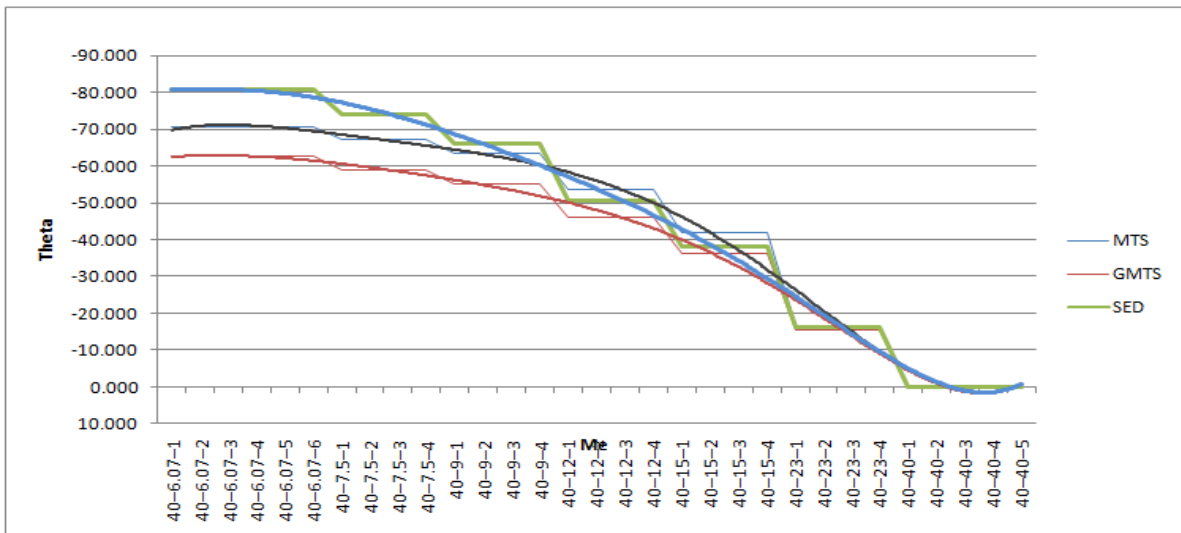


Fig 15: Comparison of prediction of crack growth direction of MTS, GMTS and SED Criterion

V. CONCLUSION

- ❖ The FEM in general and ANSYS software in particular is demonstrated in this study to provide accurate mixed mode stress intensity factors for an edge cracked semi circular disc in three point bending.
- ❖ The Strain Energy Density theory of fracture is successfully used to predict fracture surface and crack growth directions for an edge cracked semi circular disc in three point bending.
- ❖ The fracture test results for an edge cracked semi circular disc in three point bending made of PMMA are used to verify the above predictions.
- ❖ It is fair to conclude that a good agreement exists between the prediction and test results

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