ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 8, Issue 2, pp: (59-73), Month: April - June 2020, Available at: www.researchpublish.com

INTUITIONISTIC FUZZY G*S-CLOSED SETS

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Abstract: **The concept of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of fuzzy sets. In 1997 Coker introduced the concept of intuitionistic fuzzy topological spaces. In 2008, Thakur and Chaturvedi introduced the notion of intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. After that different mathematicians worked and studied in different forms of intuitionistic fuzzy g-closed set and related topological properties. The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy g*s- closed sets in intuitionistic fuzzy topological space. The class of all intuitionistic fuzzy g*s--closed sets lies between the class of all intuitionistic fuzzy semi-closed sets and class of all intuitionistic fuzzy gspr-closed sets. We also introduce the concepts of intuitionistic fuzzy g*s-- open sets, intuitionistic fuzzy g*s-continuous mappings, intuitionistic fuzzy T*s -space, intuitionistic fuzzy T**s -space and intuitionistic fuzzy *Ts space in intuitionistic fuzzy topological spaces.**

Keywords: **Intuitionistic fuzzy g*s-closed sets, Intuitionistic fuzzy g*s-open sets, Intuitionistic fuzzy g*s continuous mappings, intuitionistic fuzzy T*s -space , and intuitionistic fuzzy T**s –space.**

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [28] in 1965 and fuzzy topology by Chang [7] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [9] introduced the concept of intuitionistic fuzzy topological spaces. In 2008, Thakur and Chtuvedi [19] introduced the concepts of intuitionistic fuzzy generalized closed sets in intuitionistic fuzzy topology. After that many weak and strong forms of intuitionistic fuzzy g-closed sets such as intuitionistic fuzzy rg-closed sets[20], intuitionistic fuzzy sg-closed sets[21], intuitionistic fuzzy g*-closed sets[8], intuitionistic fuzzy gα-closed sets[13], intuitionistic fuzzy w-closed sets[22], intuitionistic fuzzy rw-closed sets[23], intuitionistic fuzzy gpr-closed sets[24], intuitionistic fuzzy rgα-closed sets[25], intuitionistic fuzzy gsp-closed sets[17], intuitionistic fuzzy gp-closed set[15] , intuitionistic fuzzy strongly g*-closed sets [2], intuitionistic fuzzy sgp-closed sets[3] , ,intuitionistic fuzzy rgw-closed sets[4] and intuitionistic fuzzy g*p-closed sets[5] have been appeared in the literature. In the present paper we introduce the concepts of g*s-closed sets due to Pushpalatha and Anitha K [14] in intuitionistic fuzzy topological spaces. The class of intuitionistic fuzzy g*s-closed sets is properly placed between the class of intuitionistic fuzzy semi closed sets and intuitionistic fuzzy gspr-closed sets. We also introduced the concepts of intuitionistic fuzzy g*s-open sets, and obtain some of their characterization and properties. As an application of this set we introduce intuitionistic fuzzy –T^*_s-space and intuitionistic fuzzy T^{**}_s-space. Further, we introduce intuitionistic fuzzy g*s -continuous mappings with some of its properties.

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II. PRELIMINARIES

Let X be a nonempty fixed set. An intuitionistic fuzzy set[1] A in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\Upsilon_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$. The intuitionistic fuzzy sets $\mathbf{0} = \{< x, 0, 1 > : x \in X\}$ and $\mathbf{1} = \{< x, 1, 0 > : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an intuitionistic fuzzy set $B = \{< x, \mu_B(x), \gamma_B(x) > : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x) > : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{< x, \gamma_A(x), \mu_A(x) >: x \in X\}$.The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{< x, \mu_{Ai}(x), \gamma_{Ai}(x) > : x \in X, (i \in \wedge)\}$ of X be the intuitionistic fuzzy set $\cap A_i = \{< x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x) > : x \in X\}$ (resp. $\cup A_i = \{< x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x) >: x \in X\}$). Two intuitionistic fuzzy sets $A = \{< x, \mu_A(x), \gamma_A(x) > : x \in X\}$ and $B = \{< x, \mu_B(x), \gamma_B(x) > : x \in X\}$ are said be q-coincident (A_qB for short) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\Im$ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [9] on X if the intuitionistic fuzzy sets $\mathbf{0}, \mathbf{1} \in \Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair $(X,\Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set .The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted int(A) [9].

Lemma 2.1 [9]: Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space $(X, \Im)$. Then:

(a) $(A_qB) \Leftrightarrow A \subseteq B^c$.

(b) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl (A) = A

(c) A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ int (A) = A.

(d) $cl(A^c) = (int(A))^c$.

(e) $int(A^c) = (cl(A))^c$.

Definition 2.1 [10]: Let X is a nonempty set and $c \in X$ a fixed element in X. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha+\beta \le 1$ then:

(a) $c(\alpha,\beta) = < x, c_\alpha, c_{1-\beta} >$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of $c(\alpha,\beta)$, and β denotes the degree of non membership of $c(\alpha,\beta)$.

(b) $c(\beta) = < x, 0, 1-c_{1-\beta} >$ is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non membership of $c(\beta)$.

Definition 2.2[11]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) An intuitionistic fuzzy semi open of X if there is an intuitionistic fuzzy set O such that $O \subseteq A \subseteq cl(O)$.

(b) An intuitionistic fuzzy semi closed if the compliment of A is an intuitionistic fuzzy semi open set.

(c) An intuitionistic fuzzy regular open of X if int(cl(A)) = A .

(d) An intuitionistic fuzzy regular closed of X if cl(int(A)) = A.

(e) An intuitionistic fuzzy pre open if $A \subseteq int(cl(A))$.

(f) An intuitionistic fuzzy pre closed if $cl(int(A)) \subseteq A$.

(g) An intuitionistic fuzzy α-open $A \subseteq int(cl(intA))$.

(h) intuitionistic fuzzy α- closed if $cl(int(cl(A))) \subseteq A$.

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Definition 2.3[11]: If A is an intuitionistic fuzzy set in intuitionistic fuzzy topological space$(X,\Im)$ then

(a) scl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy semi closed}

(b) pcl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy pre closed}

(c) αcl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy α closed}

(d) spcl(A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy semi pre-closed}

Definition 2.4: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) Intuitionistic fuzzy g-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[19]

(b) Intuitionistic fuzzy rg-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[20]

(c) Intuitionistic fuzzy sg-closed if scl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[21]

(d) Intuitionistic fuzzy g*-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g-open.[8]

(e) Intuitionistic fuzzy gα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy α- open.[13]

(f) Intuitionistic fuzzy w-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[22]

(g) Intuitionistic fuzzy rw-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular semi open.[23]

(h) Intuitionistic fuzzy gpr-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[24]

(i) Intuitionistic fuzzy rgα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular α-open.[25]

(j) Intuitionistic fuzzy gsp-closed if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy - open.[17]

(k) Intuitionistic fuzzy gp-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[15]

(l) Intuitionistic fuzzy gs-closed if scl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[18]

(m) Intuitionistic fuzzy strongly g*-closed set if cl (int(A))$\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g-open in X.[2].

(n) Intuitionistic fuzzy gspr-closed set if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – regular open.[16]

(o) Intuitionistic fuzzy sgp-closed set if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – semi open.[3]

(p) Intuitionistic fuzzy rgw-closed set if cl (intA)) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – regular semi open.[4]

(q) Intuitionistic fuzzy pre semi closed if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – g-open.[6]

(r) Intuitionistic fuzzy αgs-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[12]

(s) Intuitionistic fuzzy g*p-closed set if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g- open.[5]

The complements of the above mentioned closed set are their respective open sets.

Remark 2.1:

(a) Every intuitionistic fuzzy sg- closed set is intuitionistic fuzzy gs-closed but its converse may not be true.[18]

(b) Every intuitionistic fuzzy gs- closed set is intuitionistic fuzzy gspr-closed but its converse may not be true.[16]

(c) Every intuitionistic fuzzy gsp- closed set is intuitionistic fuzzy gspr-closed but its converse may not be true.[16]

(d) Every intuitionistic fuzzy αgs- closed set is intuitionistic fuzzy gsp-closed but its converse may not be true.[12]

Definition 2.5 [11]: Let X and Y are two nonempty sets and f : X → Y is a function. :

(a) If B = {<y, μ_B(y), γ_B(y)> : y ∈ Y}is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by f^{-1}(B), is the intuitionistic fuzzy set in X defined by

$f^{-1}(B) = \langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(b) If A = {<x, λ_A(x), ν_A(x)> : x ∈ X}is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by $f(A) = \{\langle y, f(\lambda_A)(y), f(\nu_A)(y)\rangle : y \in Y\}$

Where $f(\nu_A) = 1 - f(1- \nu_A)$.

Definition 2.6[11]: Let (X,ℑ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: X→Y be a function. Then f is said to be :

(a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.

(b) Intuitionistic fuzzy semi continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy semi open set in X.

(c) Intuitionistic fuzzy α- continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy α-open set in X.

Definition 2.7: Let (X,ℑ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: X → Y be a function. Then f is said to be:

(a) Intuitionistic fuzzy sg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy sg –closed in X.[26]

(b) Intuitionistic fuzzy rgα-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rgα –closed in X.[27]

(c) Intuitionistic fuzzy gs-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gs –closed in X.[18]

(d) Intuitionistic fuzzy gsp-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gsp –closed in X.[17]

Remark 2.2

(a) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy α-continuous, but the converse may not be true [11].

(b) Every intuitionistic fuzzy α-continuous mapping is intuitionistic fuzzy semi-continuous, but the converse may not be true [11].

(c) Every intuitionistic fuzzy semi- continuous mapping is intuitionistic fuzzy sg-continuous, but the converse may not be true [26].

(d) Every intuitionistic fuzzy sg-continuous mapping is intuitionistic fuzzy gs-continuous, but the converse may not be true[18]

(e) Every intuitionistic fuzzy gs-continuous mapping is intuitionistic fuzzy gsp-continuous, but the converse may not be true[17]

(f) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy rgα-continuous, but the converse may not be true [27].

Definition 2.8: An intuitionistic fuzzy topological space (X,ℑ) is said to be :

(a) Intuitionistic fuzzy $T_{1/2}$ space if every intuitionistic fuzzy g-closed set is closed in (X,ℑ).[19]

(b) Intuitionistic fuzzy semi $T_{1/2}$ space if every intuitionistic fuzzy sg-closed set is semi closed in (X,ℑ).[21]

(c) Intuitionistic fuzzy semi pre $T_{1/2}$ space if every intuitionistic fuzzy gsp-closed set is intuitionistic semi pre closed in (X,ℑ).[17]

(d) Intuitionistic fuzzy semi pre regular -$T_{1/2}$ space if every intuitionistic fuzzy gspr-closed set is closed in (X,ℑ).[16]

(e) Intuitionistic fuzzy gs-$T_{1/2}$ space if every intuitionistic fuzzy gs-closed set is closed in (X,ℑ).[18]

(f) Intuitionistic fuzzy rgα -$T_{1/2}$ space if every intuitionistic fuzzy rgα-closed set is closed in (X,ℑ).[25]

III. INTUITIONISTIC FUZZY G*S -CLOSED SET

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ) is called an intuitionistic fuzzy g*s -closed if scl (A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy gs-open in X.

First we prove that the class of intuitionistic fuzzy g*s- closed sets properly lies between the class of intuitionistic fuzzy semi closed sets and the class of intuitionistic fuzzy gspr-closed sets.

Theorem 3.1: Every intuitionistic fuzzy semi closed set is intuitionistic fuzzy g*s-closed.

Proof: Let A is intuitionistic fuzzy semi closed set. Let A ⊆ U and U is intuitionistic fuzzy gs-open sets in X. Since A is intuitionistic fuzzy semi closed set we have A= scl(A). Hence scl(A) ⊆ U , whenever A ⊆ U and U is intuitionistic fuzzy gs open in X. Therefore A is intuitionistic fuzzy g*s-closed set.

Remark 3.1: The converse of above theorem need not be true as from the following example.

Example 3.1: Let X = {a, b, c} and intuitionistic fuzzy sets O and U are defined as follows

O = { < a,0.9,0.1.> ,<b,0,1>,<c,0,1>}

U = { < a,0,1,> ,<b,0.8,0.1>,<c,0.6,0.3>}

Let ℑ = {***0***, O, U, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.7, 0.2>, <b ,0.6, 0.3>, <c, 0, 1> } is intuitionistic fuzzy g*s -closed but it is not intuitionistic fuzzy semi-closed .

Theorem 3.2: Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy g*s-closed.

Proof: since every intuitionistic fuzzy α-closed is intuitionistic fuzzy semi closed and by theorem 3.1 Proof follows immediately.

Remark 3.2: The converse of above theorem need not be true as from the following example.

Example 3.2: Let X = {a, b, c} and intuitionistic fuzzy sets O and U are defined as follows

O = { < a,0.6,0.2.> ,<b,0,1>,<c,0,1>} U = { < a,0.5,0.4,> ,<b,0.8,0.1>,<c,0,1>}

Let ℑ = {***0***, O, U, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.7, 0.2>,<b ,0, 1> <c, 0.7, 0.2>} is intuitionistic fuzzy g*s -closed but it is not intuitionistic fuzzy α-closed .

Theorem 3.3: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g*s-closed.

Proof: Let A is intuitionistic fuzzy closed set. Let A ⊆ U and U is intuitionistic fuzzy gs-open sets in X. Since A is intuitionistic fuzzy -closed set we have A= cl(A). But scl(A) ⊆ cl(A)., therefore scl(A) ⊆ U. whenever A ⊆ U and U is intuitionistic fuzzy gs-open in X. Hence A is intuitionistic fuzzy g*s-closed set.

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 8, Issue 2, pp: (59-73), Month: April - June 2020, Available at: www.researchpublish.com

Remark 3.3: The converse of above theorem need not be true as from the following example.

Example 3.3: Let X = {a, b} and intuitionistic fuzzy sets O and U are defined as follows

O = { < a,0.2,0.8 > ,< b,0.1,0.9>} U = { < a,0.5,0.4 > ,<b,0.6,0.4 >}

Let $\Im$ = {***0***, O, U, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.9, 0.1>,<b ,0.6, 0.3> } is intuitionistic fuzzy g*s -closed but it is not intuitionistic fuzzy closed .

Theorem 3.4: Every intuitionistic fuzzy regular -closed set is intuitionistic fuzzy g*s-closed.

Proof: It follows from the fact that every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set and Theorem 3.3.

Remark 3.4: The converse of above theorem need not be true as from the following example.

Example 3.4: Let X = {a, b, c} and intuitionistic fuzzy sets O and U are defined as follows

O = { < a,0.2,0.8 > ,< b,0.1,0.9 >, < c,0,1 >} U = { < a,0.5,0.4 > ,<b,0, 1 >,< c,0.4.,0.3 >}

Let $\Im$ = {***0***, O, U, ***1***}be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {< a, 0.9, 0.1 >,<b ,0.6, 0.3 > ,< c,0,1 > } is intuitionistic fuzzy g*s -closed but it is not intuitionistic fuzzy regular-closed .

Theorem 3.5: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy gs-closed.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy gs- open sets, U is intuitionistic fuzzy gs- open sets such that A $\subseteq$ U. Now by definition of intuitionistic fuzzy g*s-closed sets scl(A) $\subseteq$ U. We have scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy open in X. Therefore A is intuitionistic fuzzy gs-closed set.

Remark 3.5: The converse of above theorem need not be true as from the following example.

Example 3.5: Let X = {a, b, c} and intuitionistic fuzzy sets O is defined as follows

O = { < a,0.6,0.2 > ,< b,0,1 >,<c,0,1 >}

Let $\Im$ = {***0***, O, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.7, 0.2>, <b, 0.7,0.3> .<c, 0, 1>} is intuitionistic fuzzy gs -closed but it is not intuitionistic fuzzy g*s-closed .

Theorem 3.6: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy sg-closed.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy semi open sets in X. Since every intuitionistic fuzzy semi open set is intuitionistic fuzzy gs- open sets, U is intuitionistic fuzzy gs- open sets such that A $\subseteq$ U. Now by definition of intuitionistic fuzzy g*s-closed sets, scl(A) $\subseteq$ U. We have scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy semi open in X. Therefore A is intuitionistic fuzzy sg-closed set.

Remark 3.6: The converse of above theorem need not be true as from the following example.

Example 3.6: Let X = {a, b, c, d} and intuitionistic fuzzy sets O,U, V and W are defined as follows

O = { < a,0.9 ,0.1 > ,<b,0, 1>,<c,0, 1>,<d, 0, 1>}

U = { < a ,0, 1> ,<b, 0.8, 0.1>,<c, 0, 1>,<d ,0, 1>}

V = { < a, 0.9, 0.1> ,<b, 0.8 ,0.1 >,<c, 0, 1>,<d, 0, 1>}

W = { < a, 0.9, 0.1> ,<b, 0.8 ,0.1>,<c ,0.7, 0.2>,<d, 0, 1>}

Let ℑ = {***0***, O, U**,** V, W, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {< a , 0, 1 >,< b, 0, 1> < c, 0.7, 0.2 >,< d ,0, 1 > } is intuitionistic fuzzy sg-closed set but it is not intuitionistic fuzzy g*s-closed.

Theorem 3.7: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy gspr-closed.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A ⊆ U and U is intuitionistic fuzzy regular open sets in X. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy gs- open sets /, U is intuitionistic fuzzy gs- open sets such that A ⊆ U. Now by definition of intuitionistic fuzzy g*s-closed sets scl(A) ⊆ U. Note that spcl(A) ⊆ scl(A) is always true. We have spcl(A) ⊆ U whenever A ⊆ U and U is intuitionistic fuzzy regular open in X. Therefore A is intuitionistic fuzzy gspr-closed set.

Remark 3.7: The converse of above theorem need not be true as from the following example.

Example 3.7: Let X = {a, b, c, d} and intuitionistic fuzzy sets O and U are defined as follows

O = { < a, 0.7, 0.1> ,<b, 0.6, 0.2 >,<c ,0, 1>,<d, 0, 1 > }

U = { < a, 0, 1> ,<b, 0, 1>,<c, 0.7, 0.2>,< d, 0.6, 0.1 > }

Let ℑ = {***0***, O, U**,** ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A ={< a, 0.7, 0.1 > ,< b, 0, 1 >,< c, 0, 1 > ,< d, 0, 1 >} is intuitionistic fuzzy gspr -closed but it is not intuitionistic fuzzy g*s-closed.

Theorem 3.8: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy gsp-closed.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A ⊆ U and U is intuitionistic fuzzy open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy gs-open, U is intuitionistic fuzzy gs-open set Now by definition of intuitionistic fuzzy g*s-closed sets scl(A) ⊆ U. Note that spcl(A) ⊆ scl(A) is always true. We have spcl(A) ⊆ U whenever A ⊆ U and U is intuitionistic fuzzy open in X. Therefore A is intuitionistic fuzzy gsp-closed set.

Remark 3.8: The converse of above theorem need not be true as from the following example.

Example 3.8: Let X = {a, b} and ℑ = {***0***, U**,** ***1***}be an intuitionistic fuzzy topology on X, where U={<a, 0.5, 0.3>,<b, 0.2, 0.3>}.Then the intuitionistic fuzzy set A = {< a, 0.2 ,0.4 >,< b, 0.6, 0.1 >} is intuitionistic fuzzy gsp -closed but it is not intuitionistic fuzzy g*s- closed.

Theorem 3.9: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy αgs-closed.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A ⊆ U and U is intuitionistic fuzzy semi open sets in X. Since every intuitionistic fuzzy semi open set is intuitionistic fuzzy gs-open, U is intuitionistic fuzzy gs-open set Now by definition of intuitionistic fuzzy g*s-closed sets ,scl(A) ⊆ U. Note that αcl(A) ⊆ scl(A) is always true. We have αcl(A) ⊆ U whenever A ⊆ U and U is intuitionistic fuzzy semi open in X. Therefore A is intuitionistic fuzzy αgs-closed set.

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 8, Issue 2, pp: (59-73), Month: April - June 2020, Available at: www.researchpublish.com

Remark 3.9: The converse of the above theorem need not be true as from the following example.

Example 3.9: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U and V are defined as follows

O = { < a,0.9,0.i> ,<b,0,1>,<c,0,1>,<d,0,1>}

U = { < a,0,1,> ,<b,0.8,0.1>,<c,0,1>,<d,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1,>,<c,0,1>,<d,0,1>}

Let ℑ = {***0***, O, U, V, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {< a,0, 1 >,< b, 0, 1 > ,< c, 0.7, 0.2 >,< d, 0, 1 >} is intuitionistic fuzzy αgs -closed but it is not intuitionistic fuzzy g*s-closed.

Theorem 3.10: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy pre semi -closed.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A ⊆ U and U is intuitionistic fuzzy g- open sets in X. Since every intuitionistic fuzzy g-open set is intuitionistic fuzzy gs-open, U is intuitionistic fuzzy gs-open set . Now by definition of intuitionistic fuzzy g*s-closed sets scl(A) ⊆ U. Note that spcl(A) ⊆ scl(A) is always true. We have spcl(A) ⊆ U whenever A ⊆ U and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy pre semi-closed set.

Remark 3.10: The converse of above theorem need not be true as from the following example.

Example 3.10: Let X = {a, b, c} and intuitionistic fuzzy sets O is defined as follows

O = { < a,0.6,0.2.> ,<b,0,1>,<c,0,1>}

Let ℑ = {***0***, O, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {< a, 0.7, 0.2 >, <b, 0.7, 0.3 > ,< c, 0, 1 >} is intuitionistic fuzzy pre semi -closed but it is not intuitionistic fuzzy g*s-closed .

Theorem 3.11: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy rgα -closed.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A ⊆ U and U is intuitionistic fuzzy regular α- open sets in X. Since every intuitionistic fuzzy regular α-open set is intuitionistic fuzzy semi open set and every intuitionistic fuzzy semi open is intuitionistic fuzzy gs-open, therefore U is intuitionistic fuzzy gs-open set Now by definition of intuitionistic fuzzy g*s-closed sets scl(A) ⊆ U. Note that αcl(A) ⊆ scl(A) is always true. We have αcl(A) ⊆ U whenever A ⊆ U and U is intuitionistic fuzzy regular α-open in X. Therefore A is intuitionistic fuzzy rgα-closed set

Remark 3.11: The converse of the above Theorem need not be true, as seen from the following example:

Example 3.1: Let *X* = {*a*, *b*, *c*, *d*, *e*} and intuitionistic fuzzy sets *P*, *Q*, *R*, *S*, *T*, *U* and *V* are defined as follows:

P = {< *a*,0.9,0.1>,<*b*,0,1>,<*c*,0,1>,<*d*,0,1> <*e*,0,1>}

Q = {< *a*,0,1>,<*b*,0,1>,<*c*,0,1>,<*d*,0.9,0.1> <*e*,0,1>}

R = {< *a*,0,1>,<*b*,0,1>,<*c*,0,1>,<*d*,0,1>,<*e*,0.8,0.2>}

S = {< *a*,0.9,0.1>,<*b*,0,1>,<*c*,0,1>,<*d*,0.9,0.1> <*e*,0,1>}

T = {< *a*,0.9,0.1> ,<*b*,0,1>,<*c*,0,1>,<*d*,0,1 > <*e*,0.8,0.2>

U = {< *a*,0,1> ,<*b*,0,1>,<*c*,0,1>, <*d*,0.9,0.1>,<*e*,0.8,0.2>}

V ={<*a*,0.9,0.1>,<*b*,0,1>,<*c*,0,1>, <*d*,0.9,0.1>, <*e*,0.8,0.2>}

Let Á = {***0*** , *P*, *Q*, *R*, *S*, *T*, *U*, *V*, ***1*** } be an intuitionistic fuzzy topology on *X*. Then the intuitionistic fuzzy set *A* = {<*a*,0,1>,<*b*,0.9,0.1>,<*c*,0,1>,<*d*,0,1>,<*e*,0,1>} is intuitionistic fuzzy *rg*a -closed but it is not intuitionistic fuzzy g*s closed.

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 8, Issue 2, pp: (59-73), Month: April - June 2020, Available at: www.researchpublish.com

Remark 3.12: From the above discussion and known results we have the following diagram of implications:

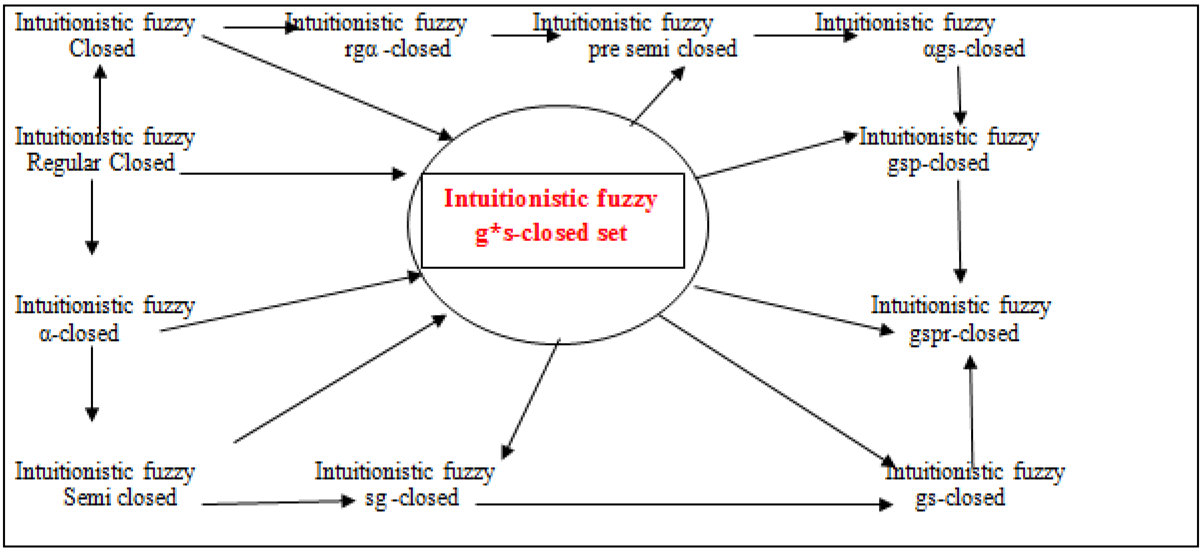

Fig 1: Relations between intuitionistic fuzzy g*s-closed set and other existing intuitionistic fuzzy closed sets

Theorem 3.12: Let $(X,\Im)$ be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy g*s-closed closed if and only if $\rceil(AqF) \Rightarrow \rceil(scl(A)qF))$ for every intuitionistic fuzzy gs- closed set F of X.

Proof: Necessity: Let F be an intuitionistic fuzzy gs-closed set of X and $\rceil(AqF)$. Then by Lemma 2.1(a), $A \subseteq F^c$ and F^c is intuitionistic fuzzy gs-open in X. Therefore $scl(A) \subseteq F^c$ by Def 3.1 because A is intuitionistic fuzzy g*s-closed. Hence by lemma 2.1(a), $\rceil(scl(A)qF))$.

Sufficiency: Let O be an intuitionistic fuzzy gs-open set of X such that $A \subseteq O$ i.e. $A \subseteq ((O)^c)^c$ Then by Lemma 2.1(a), $\rceil(A_q O^c)$ and O^c is an intuitionistic fuzzy gs- closed set in X. Hence by hypothesis $\rceil(scl\ (A)\ _q\ O^c)$. Therefore by Lemma 2.1(a), $scl\ (A) \subseteq ((O)^c)^c$ i .e. $scl\ (A) \subseteq O$ Hence A is intuitionistic fuzzy g*s-closed in X.

Remark 3.13: The intersection of two intuitionistic fuzzy g*s-closed sets in an intuitionistic fuzzy topological space $(X,\Im)$ may not be intuitionistic fuzzy g*s-closed. For,

Example 3.12: Let X = {a, b, c, d} and intuitionistic fuzzy sets O,U, V and W are defined as follows

O = { < a ,0.9, 0.1 > ,<b, 0 ,1>,<c, 0, 1>,<d, 0 ,1>}

U = { < a, 0, 1,> ,<b ,0.8 ,0.1>,<c ,0, 1> ,<d ,0, 1>}

V = { < a ,0.9, 0.1> ,<b, 0.8, 0.1,>,<c, 0, 1>,<d, 0, 1>}

W = { < a, 0.9, 0.1,> ,<b,0.8, 0.1 >,<c, 0.7, 0.2>,< d, 0, 1>}

Let $\Im = \{0, O, U, V, W, 1\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.9, 0.1 >,<b, 0.8, 0.1 > < c, 0 ,1>,< d, 0, 1>} and

B = {<a, 0.9, 0.1 >, < b, 0, 1>, < c, 0.7, 0.2 > ,< d, 0.9 ,0.1>}are intuitionistic fuzzy g*s-closed in $(X, \Im)$ but $A \cap B$ is not intuitionistic fuzzy g*s-closed.

Theorem 3.13: Let A be an intuitionistic fuzzy g*s-closed set in an intuitionistic fuzzy topological space $(X, \Im)$ and $A \subseteq B \subseteq scl\ (A)$ .Then B is intuitionistic fuzzy g*s-closed in X.

Proof: Let O be an intuitionistic fuzzy gs-open set in X such that $B \subseteq O$. Then $A \subseteq O$ and since A is intuitionistic fuzzy g*s-closed, $scl(A) \subseteq O$. Now $B \subseteq scl\ (A) \Rightarrow scl\ (B) \subseteq scl\ (A) \subseteq O$.. Consequently B is intuitionistic fuzzy g*s-closed.

Definition 3.2: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, ℑ) is called intuitionistic fuzzy g*s--open if and only if its complement A^c is intuitionistic fuzzy g*s-closed.

Remark 3.14*:* Every intuitionistic fuzzy open set is intuitionistic fuzzy g*s-open but its converse may not be true.

Example 3.13: Let X = {a, b} and ℑ = {***0***,U, ***1***}be an intuitionistic fuzzy topology on X, where U={< a ,0.7, 0.2 >,< b ,0.6 ,0.3>}.Then the intuitionistic fuzzy set A={<a, 0.2, 0.7 >,< b, 0.1, 0.8 >} is intuitionistic fuzzy g*s-open in (X, ℑ) but it is not intuitionistic fuzzy open in (X, ℑ).

Theorem 3.14: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy g*s-open if $F \subseteq$ scl(A) whenever F is intuitionistic fuzzy gs-closed and $F \subseteq A$.

Proof: Follows from definition 3.1 and Lemma 2.1.

Theorem 3.15: Let A be an intuitionistic fuzzy g*s-open set of an intuitionistic fuzzy topological space (X ,ℑ) and sint (A) $\subseteq B \subseteq A$. Then B is intuitionistic fuzzy g*s-open.

Proof: Suppose A is an intuitionistic fuzzy g*s-open in X and sint(A) $\subseteq B \subseteq A$. $\Rightarrow A^c \subseteq B^c \subseteq (\text{sint}(A))^c \Rightarrow A^c \subseteq B^c \subseteq \text{scl}(A^c)$ by Lemma 2.1(d) and A^c is intuitionistic fuzzy g*s-closed it follows from theorem 3.13 that B^c is intuitionistic fuzzy g*s-closed .Hence B is intuitionistic fuzzy g*s-open.

IV. INTUITIONISTIC FUZZY G*S- CONTINUOUS MAPPINGS

Definition 4.1: A mapping *f*: (X, ℑ) →(Y, σ) is intuitionistic fuzzy g*s- continuous if inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy g*s-closed set in X.

Theorem 4.1: A mapping *f* : (X, ℑ)→(Y, σ) is intuitionistic fuzzy g*s- continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy g*s- open in X.

Proof*:* It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.

Remark4.1: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g*s-continuous, but converse may not be true. For,

Example 4.1: Let X = {a, b}, Y ={x, y} and intuitionistic fuzzy sets U and V are defined as follows :

U= {< a, 0.7, 0.2 >, < b, 0.6, 0.3 >} , V= {< x, 0.7, 0.2>, <y, 0.8, 0.1>}

Let ℑ = {***0***, U, **1**} and σ = {**0**, V, **1**} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,ℑ) →(Y, σ) defined by *f* (a) = x and *f* (b) = y is intuitionistic fuzzy g*s- continuous but not intuitionistic fuzzy continuous.

Remark4.2: Every intuitionistic fuzzy semi continuous mapping is intuitionistic fuzzy g*s-continuous, but converse may not be true. For,

Example 4.2: Let X = {a, b, c}, Y ={x, y, z} and intuitionistic fuzzy sets U and V are defined as follows :

U= {< a, 0.7, 0.2>, < b, 0.6, 0.3>, < c, 0.4, 0.4 > } V= {< x, 0.6, 0.3 >, <y, 0.8, 0.2>, < z,0.6,0.2 >}

Let ℑ = {***0***, U, **1**} and σ = {**0**, V, **1**} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,ℑ) →(Y, σ) defined by *f* (a) = x , *f* (b) = y and *f*(c)= z is intuitionistic fuzzy g*s- continuous but not intuitionistic fuzzy semi continuous.

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 8, Issue 2, pp: (59-73), Month: April - June 2020, Available at: www.researchpublish.com

Remark4.3: Every intuitionistic fuzzy g*s-continuous mapping is intuitionistic fuzzy gs-continuous, but converse may not be true. For,

Example 4.3: Let X = {a, b}, Y ={x ,y} and intuitionistic fuzzy sets U and V are defined as follows :

U= {< a, 0.5, 0.5>, < b, 0.4, 0.6>}, V= {< x, 0.5, 0.5>, < y 0.3, 0.7>}

Let $\mathfrak{I}$ = { ***0,*** U, ***1***} and σ ={ **0,** V, **1**} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X, $\mathfrak{I}$) →(Y, σ) defined by *f* (a) = x and *f* (b) = y is intuitionistic fuzzy gs- continuous but not intuitionistic fuzzy g*s-continuous.

Remark 4.4: Every intuitionistic fuzzy g*s-continuous mapping is intuitionistic fuzzy gsp-continuous, but converse may not be true. For,

Example 4.3: Let X = {a, b, c, d, e} and Y= {p, q, r, s, t} and intuitionistic fuzzy sets O, U, V and W are defined as follows

O = { < a ,0.9 ,0.1 > ,< b,0.8, 0.1>,< c, 0, 1 >,< d, 0, 1 > < e,0,1>}

U = { < a,0, 1 > ,< b,0,1 >,< c,0.8,0.1 >,< d, 0.7,0.2 > < e,0, 1>}

V = { < a,0.9,0.1> ,< b,0.8,0.1 >,< c,0.8,0.1 >, < d,0.7,0.2 >,< e, 0,1 >}

W= {< p,0.9,0.1 >,< q,0,1 >,< r, 0, 1> ,< s,0,1 >,< t, 0 ,1 >}

Let $\mathfrak{I}$ = {***0***, O, U, V ***, 1***}and σ ={***0***, W, ***1***}be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X, $\mathfrak{I}$) →(Y, σ) defined by *f* (a) = p, *f* (b) = q, *f* (c) = r, *f* (d) = s and *f* (e) = t is intuitionistic fuzzy gsp- continuous but not intuitionistic fuzzy g*s- continuous.

Remark 4.5: Every intuitionistic fuzzy g*s-continuous mapping is intuitionistic fuzzy sg-continuous, but converse may not be true. For,

Example 4.5: Let X = {a, b, c}, Y ={x, y, z} and intuitionistic fuzzy sets O, U and V are defined as follows

O = {< a, 0.9, 0.1 >,< b, 0,1 > < c, 0,1 >}

U= {< a, 0, 1 > ,<b, 0.8, 0.1 > ,< c, 0,1> , V = {< x, 0.9, 0.1>, <y,0.8,0.1> ,<z,0,1>}

Let $\mathfrak{I}$ = {***0***, O, U, ***1***} and σ ={{***0***, V ***,1***}be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X, $\mathfrak{I}$) →(Y, σ) defined by *f* (a) = x *f* (b) = y and *f* (c) = z is intuitionistic fuzzy sg-continuous but it is not intuitionistic fuzzy g*s- continuous.

Remark 4.6: Every intuitionistic fuzzy g*s-continuous mapping is intuitionistic fuzzy rgα-continuous, but converse may not be true. For,

Example 4.6: Let X = {a, b, c, d, e} and Y= {w, x, y, z, t} and intuitionistic fuzzy sets P, Q, R, S, T, U, V and W defined as follows

P = { < a,0.9,0.1 > ,<b, 0, 1 >,< c, 0, 1 >,< d,0, 1 > < e,0, 1 >}

Q = { < a,0,1 > ,< b,0,1 >,< c,0,1 >,< d,0.9, 0.1 > < e,0,1 >}

R = { < a,0,1 > ,< b ,0,1 >,< c, 0, 1 >, < d,0, 1 >,< e,0.8, 0.2 >}

S = { < w, 0.9, 0.1 > ,< x, 0,1 >,< y, 0, 1 >,< z, 0.9, 0.1 > < t, ,0, 1>}

Let $\mathfrak{I}$ = {**0**, P, Q, R, ***1***} and σ = {***0,*** S, ***1***} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\mathfrak{I}$) →(Y, σ) defined by *f* (a) = w, *f* (b) = x, *f* (c) = y, *f* (d) = z and *f* (e) = t is intuitionistic fuzzy rgα-continuous but not intuitionistic fuzzy g*s-continuous.

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 8, Issue 2, pp: (59-73), Month: April - June 2020, Available at: www.researchpublish.com

Remark 4.7: From the above discussion and known results we have the following diagram of implication

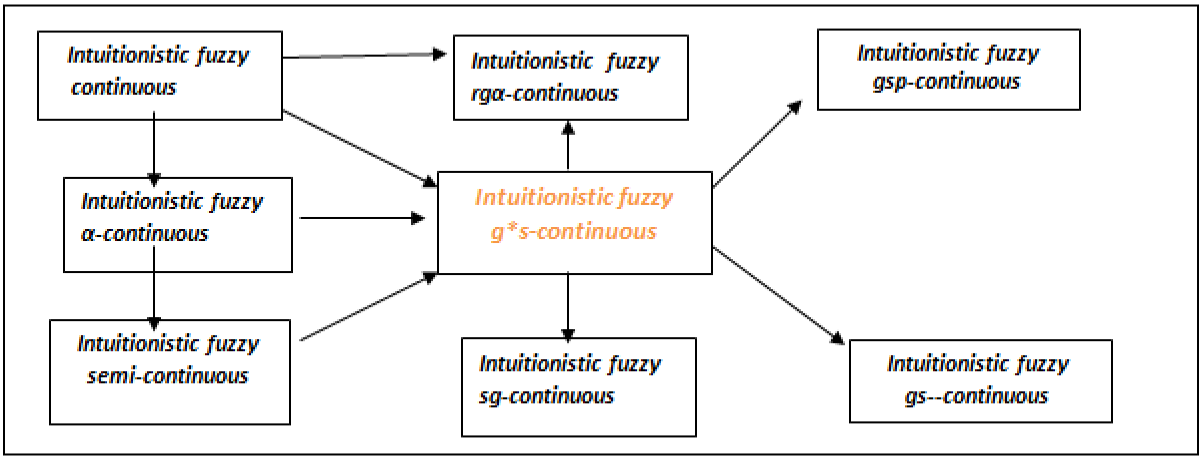

Fig 2: Relations between intuitionistic fuzzy g*s-continuous mappings and other existing intuitionistic fuzzy continuous mappings.

Theorem 4.2*:* If f: $(X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g*s- continuous then for each intuitionistic fuzzy point c(α,β) of X and each intuitionistic fuzzy gs-open set V of Y such that $f(c(\alpha,\beta)) \subseteq V$ there exists an intuitionistic fuzzy g*s- open set U of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) \subseteq V$

Proof *:* Let c(α,β) be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy gs-open set of Y such that $f(c(\alpha,\beta)) \subseteq V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy g*s-open set of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Theorem 4.3: Let f: $(X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g*s- continuous then for each intuitionistic fuzzy point c(α,β) of X and each intuitionistic fuzzy open set V of Y such that .$f(c(\alpha,\beta))qV$, there exists an intuitionistic fuzzy g*s- open set U of X such that $c(\alpha,\beta)qU$ and $f(U) \subseteq V$.

Proof: Let c(α,β) be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that $f(c(\alpha,\beta))q V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy g*s- open set of X such that $c(\alpha,\beta)q U$ and $f(U)= f(f^{-1}(V)) \subseteq V$.

Theorem 4.4: If f: $(X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g*s-continuous and g :$(Y, \sigma) \rightarrow (Z, \mu)$ is intuitionistic fuzzy continuous. Then gof : $(X,\Im) \rightarrow (Z, \mu)$ is intuitionistic fuzzy g*s-continuous**.**

Proof: Let A is an intuitionistic fuzzy closed set in Z. then $g^{-1}(A)$ is intuitionistic fuzzy closed in Y because g is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy g*s – closed in X. Hence gof is intuitionistic fuzzy g*s –continuous.

Theorem 4.5: If f : $(X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g*s-continuous and g : $(Y, \sigma) \rightarrow (Z,\mu)$ is intuitionistic fuzzy gs-continuous and (Y, σ) is intuitionistic fuzzy gs -$T_{1/2}$ –space then gof : $(X,\Im) \rightarrow (Z,\mu)$ is intuitionistic fuzzy g*s-continuous.

Proof: Let A is an intuitionistic fuzzy closed set in Z, then $g^{-1}(A)$ is intuitionistic fuzzy gs-closed in Y. Since Y is intuitionistic fuzzy gs- $T_{1/2}$-space, then $g^{-1}(A)$ is intuitionistic fuzzy closed in Y. Hence $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy g*s – closed in X. Hence gof is intuitionistic fuzzy g*s – continuous.

V. APPLICATION OF INTUITIONISTIC FUZZY G*S- CLOSED SETS

In this section we introduce intuitionistic fuzzy T^*_s-space, , and intuitionistic fuzzy T^{**}_s – space as an application of intuitionistic fuzzy g*s-closed set. We have derived some characterizations of intuitionistic fuzzy g*s-closed sets.

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 8, Issue 2, pp: (59-73), Month: April - June 2020, Available at: www.researchpublish.com

Definition 5.1: An intuitionistic fuzzy topological space (X,ℑ) is called:

(i) an intuitionistic fuzzy T^*_s-space if every intuitionistic fuzzy g*s-closet set is intuitionistic fuzzy closed.

(ii) an intuitionistic fuzzy T^{**}_s -space if every intuitionistic fuzzy g*s-closet set is intuitionistic fuzzy semi closed.

Theorem 5.1: Every intuitionistic fuzzy T^*_s -space is intuitionistic fuzzy T^{**}_s - space.

Proof: Let (X, ℑ) be an intuitionistic fuzzy T^*_s space and let A be intuitionistic fuzzy g*s-closed set in (X, ℑ).Then A is intuitionistic Fuzzy -closed, Since every intuitionistic fuzzy closed set is intuitionistic fuzzy semi-closed , A is intuitionistic fuzzy semi closed in topological space (X,ℑ) Hence (X, ℑ) is intuitionistic fuzzy T^{**}_p space.

Remark 5.1: The converse of the above theorem need not be true, as seen from the following example

Example 5.1: Let X = {a, b} and Let ℑ = {**0**, O, **1**} be an intuitionistic fuzzy topology on X, where O = {<a ,0.5, 0.5 >,<b ,0.4 ,0.6>}. Then intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy T^{**}_s space but not intuitionistic fuzzy T^*_s space.

Theorem 5.2: Every intuitionistic fuzzy semi pre $T_{1/2}$ space is intuitionistic fuzzy T **s-space.

Proof: Let (X, ℑ) be an intuitionistic fuzzy semi pre $T_{1/2}$-space and let A be an intuitionistic fuzzy g*s-closed set in (X, ℑ). Since every intuitionistic fuzzy g*s closed is intuitionistic fuzzy gsp-closed Then A is intuitionistic fuzzy gsp-closed in(X, ℑ) . Now (X, ℑ) be an intuitionistic fuzzy semi pre $T_{1/2}$ -space, Then by definition of intuitionistic fuzzy semi pre $T_{1/2}$ space , A is intuitionistic fuzzy semi pre closed set in topological space (X, ℑ) .But every intuitionistic fuzzy semi pre closed set is intuitionistic fuzzy semi closed , therefore A is intuitionistic fuzzy semi open in(X, ℑ) Hence (X, ℑ) is intuitionistic fuzzy T^{**}_s- space.

Remark 5.2: The converse of the above theorem need not be true, as seen from the following example

Example 5.2: Let X = {a, b} and Let ℑ = {**0**, A, B, **1**} be an intuitionistic fuzzy topology on X where A={< a ,0.7, 0.5 >,< b ,0.3 , 0.6 >} and B ={< a ,0.7, 0.3 >,< b ,0.0 ,0.1>} .Then intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy T**s space but not intuitionistic fuzzy semi pre $T_{1/2}$ space.

Theorem 5.3: Every intuitionistic fuzzy semi pre regular $T_{1/2}$ space is intuitionistic fuzzy T^*_s space.

Proof: Let (X, ℑ) be an intuitionistic fuzzy semi pre regular $T_{1/2}$ and let A be intuitionistic fuzzy g*s-closed set in (X, ℑ). Since every intuitionistic fuzzy g*s closed is intuitionistic fuzzy gspr-closed Then A is intuitionistic fuzzy gspr-closed in(X, ℑ). Now (X, ℑ) be an intuitionistic fuzzy semi pre regular $T_{1/2}$ -space, Then by definition of intuitionistic fuzzy semi pre regular $T_{1/2}$ space, A is intuitionistic fuzzy closed set in (X, ℑ) . Hence (X, ℑ) is intuitionistic fuzzy T^*_s space.

Remark 5.3: The converse of the above theorem need not be true, as seen from the following example

Example 5.3: Let X = {a, b ,c, d } and Let ℑ = {{***0***, A, B, ***1***} be an intuitionistic fuzzy topology on X , where A={<a ,0.7, 0.5 >,<b ,0.3 ,0.6>, < c, 1, 0 > < d, 0,1>}

B={<a ,0.7, 0.3 >,<b ,0.0 ,0.1>, < c, 0, 1> <d, 0.5,0.5>} .

Then intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy T^*_s space but not intuitionistic fuzzy semi pre regular $T^*{}_{-1/2}$ space.

Theorem 5.4: Every intuitionistic fuzzy semi $T_{1/2}$ -space is intuitionistic fuzzy T^{**}_s - space.

Proof: Let (X, ℑ) be an intuitionistic fuzzy semi $T_{1/2}$-space and let A be intuitionistic fuzzy g*s-closed set in (X, ℑ). Since every intuitionistic fuzzy g*s closed is intuitionistic fuzzy sg-closed Then A is intuitionistic Fuzzy sg-closed in (X, ℑ) . Now (X, ℑ) be an intuitionistic fuzzy semi $T_{1/2}$ -space, Then by definition of intuitionistic fuzzy semi $T_{1/2}$ space , A is intuitionistic fuzzy semi closed set in (X, ℑ) . Hence (X, ℑ) is intuitionistic fuzzy T^{**}_s -space.

Remark 5.4: The converse of the above theorem need not be true, as seen from the following example

Example 5.4: Let X = {a, b} and Let ℑ = {**0**, O, **1**} be an intuitionistic fuzzy topology on X , where O={<a ,0.5, 0.5 >,<b ,0.4 ,0.6>}. Then intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy T^{**}_s - space but not intuitionistic fuzzy semi $T_{1/2}$ -space.

Theorem 5.5: Every intuitionistic fuzzy rgα -$T_{1/2}$ -space is intuitionistic fuzzy T^{*}_s space.

Proof: Let (X, ℑ) be an intuitionistic fuzzy rgα-$T_{1/2}$ space and let A be intuitionistic fuzzy g*s-closed set in (X, ℑ). Since every intuitionistic fuzzy g*s closed is intuitionistic fuzzy rgα-closed Then A is intuitionistic fuzzy rgα-closed in(X, ℑ) . Now (X, ℑ) be an intuitionistic fuzzy rgα $T_{1/2}$ -space, Then by definition of intuitionistic fuzzy rgα-$T_{1/2}$ space, A is intuitionistic fuzzy closed set in (X, ℑ) . Hence (X, ℑ) is intuitionistic fuzzy T^{*}_s space.

Remark 5.5: The converse of the above theorem need not be true, as seen from the following example

Example 5.5: Let X = {a, b,c, d} and Let ℑ = {**0**, O, **1**} be an intuitionistic fuzzy topology on X , where O={<a ,0.5, 0.5 >,<b ,0.4 ,0.6> < c,0.8,0.2,>, < d,05,0.5>}. Then intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy T^{*}_s - space but not intuitionistic fuzzy rgα -$T_{1/2}$ -space.

VI. CONCLUSION

The theory of g-closed sets plays an important role in general topology. Since its inception many weak and strong forms of g-closed sets have been introduced in general topology as well as fuzzy topology and intuitionistic fuzzy topology. The present paper investigated a new form of intuitionistic fuzzy closed sets called intuitionistic fuzzy g*s-closed sets which contain the classes of intuitionistic fuzzy closed sets, intuitionistic fuzzy semi closed sets, intuitionistic fuzzy α-closed sets, intuitionistic fuzzy regular closed, and contained in the classes of intuitionistic fuzzy gs-closed sets, intuitionistic fuzzy sg-closed sets , intuitionistic fuzzy gsp-closed sets , intuitionistic fuzzy gspr-and class of all intuitionistic fuzzy rgα-closed sets. Several properties and application of intuitionistic fuzzy g*s--closed sets and intuitionistic fuzzy g*s-continuous mappings are studied. Many examples are given to justify the result.

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