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Relationship between Parent and Daughter Spring Constants

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Abstract: **This paper presents the relationship between parent and daughter spring constants measure and analyzed for three helical springs. The study employed one group pretest-post test experimental design with neither randomization nor control group. The three helical springs A, B and C used as experimental group were subjected to a pretest determination of their spring constant before they were each cut into a section equal to half of its original length (daughter spring). A post test determination of the spring constant in each of the daughter springs was done.**

The study found out that the spring constant of parent springs A, B and C were; 43.8338 N/m, 10.0988 N/m and 12.2840 N/m respectively while as the daughter spring constants for A, B and C were 73.8149 N/m, 19.6200 N/m and 23.9443 N/m respectively.

The results of this study show that the spring constant of any spring when cut into half of its original length, is never constant but is in agreement with Thomas young`s theory.

***Keywords*: Pretest-post test experiment, Parent constant, Daughter constant.**

1. INTRODUCTION

In mechanics, a spring is an elastic object used to store mechanical energy. Simple non-coiled springs were used throughout human history for example the bow and arrow (Payson, 1998). According to Gerhard (1978) coiled springs appeared early in the fifteenth century in door locks and powered-clocks.

There are different types of springs whose design depend on the purpose they are to serve. They include; tension spring, compression springs, torsion springs, variable springs, flat springs, machined springs, constant springs and helical spring which is made by winding a wire round a cylinder and has the property of stretching or compressing when a pulling or pushing force is applied.

In 1676, Robert Hooke discovered Hooke's law which states that the force a spring exerts is proportional to its extension.

$$F \propto x \qquad 1.1$$

$$F = kx \qquad 1.2$$

Where $x = l - l_o$

k is a spring constant, giving a measure of stiffness of a spring called the restoring force per unit extension of the spring (White L, 1966). The stiffer a spring is, the more difficult it is to be compressed or stretched.

Many students and teachers do not know what happens to the spring constant when the parent spring is cut into parts (daughter spring) (Samuel A and Weir J, 1999). The few who can formulate formulas relating the spring constant (k) with the length of the spring, can easily tell that daughter spring constant is more than the parent spring constant but cannot describe the special relationship that exist between the values of the parent spring constant and the daughter spring constant as shown in the case study of the students question on physics (yahoo.com /questions, 2019).

It has always been taught that the spring constant (k) is a constant for a given spring, and that k, will always be the same regardless of what you do to the spring. For any practice problem in which a spring of length l is cut into four parts of length, $\frac{l}{4}$, the spring constant in each of the new springs cut from the old spring (K_{new}) is,

$$K_{new} = \mathrm{k} \times \frac{Original\ length}{4} \qquad 1.3$$

However, it is thought that this is false and that k will be the same even if the spring is cut into different lengths since k is an inherent property of the spring, therefore, it has to remain constant. However, if so then why. In response to this question, Parmley (2000) explained that the spring's a quarter length will have a spring constant four times as high as the original length. Sclater (2011) also supplemented on this argument with an explanation based on deductions from conservation of potential energy of the spring.

In a related circumstance, Serway and Jewett (2010) explains that if the spring is literally cut into a half of the material, wire diameter and coil diameter stay the same and only the number of active coils gets reduced by a half and therefore k in each of the daughter springs doubles.

However, Francis Bacon (1561-1626) an English Philosopher and scientist active in the 17th century disagreed with the method of answering scientific questions by deductions and advocated for experiments to test existing theories or new hypotheses to support or disprove them (Mathews .S, 2008).

Thomas young proposed that if a parent spring with spring constant k and length, m, is cut into a section of length n, the constant of the cut out section (daughter spring) is given as in equation below (Quora, 2018),

$$kx\frac{\mathrm{m}}{\mathrm{n}} \qquad 1.4$$

Provided that the other factors that affect the spring constant like material, wire and coil diameter are not changed. According to young, the nature and direction of the relationship can be presented with a mathematical model as;

$$K_{daughter} = \frac{m}{n}K_{parent} \qquad 1.5$$

However, the limitation in young's theory is that it was based on theoretical deductions using existing laws and principles with no practical work done to test its validity when a spring is cut into a section half of the original length. This necessitated this study which considered the values of the parent spring constant as independent variable and the values of the daughter spring constant as the dependent variable. The independent variable was varied by use of springs of different dimensions which yielded varied values of the parent spring constants and consequently varied values of daughter spring constants.

2. MATERIALS AND METHODS

2.1 MATERIALS

Three sample steel springs, A, B and C of different dimensions as shown in table 1 were selected. A 100 g mass hanger, 400 gram masses in the intervals of 100 grams, a clamp stand, a meter rule, a pin to act as a pointer, a rigid support, a spring cutter and a modeling clay.

Table 1: Dimensions of the sample helical steel springs

Spring	Parent Length, m, (cm)	Daughter Length, n, (cm)	Diameter of the coil (mm)	Diameter of the wire (mm)	$\frac{m}{n}$
A	32	16	11	0.59	2
B	52	26	11	0.57	2
C	62	31	7	0.56	2

2.2 METHODS

The study employed one group pretest-posttest experimental design with neither randomization nor control group. In this study the experimental groups were the sample springs A, B and C. A pretest determination of the values of the spring constant in each of the springs was done and later a treatment of cutting each of the springs into a section equal to half of its original length was done and finally a posttest determination of the spring constant in the cut out section was done to assess the effect of the treatment on the values of the spring constant.

Steel Spring A with a pointer on its lower end was hang from a rigid support, see figure 1 and with no load hooked on the spring, the initial reading of the pointer on the scale was read on the meter rule. A load of 100 gram was hooked on the spring and in a steady position, the new reading was noted and the difference in readings between the initial reading and the new reading was taken, it's the extension, x.

The load hooked on the spring was increased to 200g, 300g and 400g and the corresponding pointer readings on the scale and their extension were noted. The experiment was repeated for the parent steel springs B and C.

The procedure above was then repeated with the daughter springs of A, B and C, put on the rigid support and the corresponding, original lengths, final pointer readings and extensions got. Figure 1 shows how the experiments were carried out.

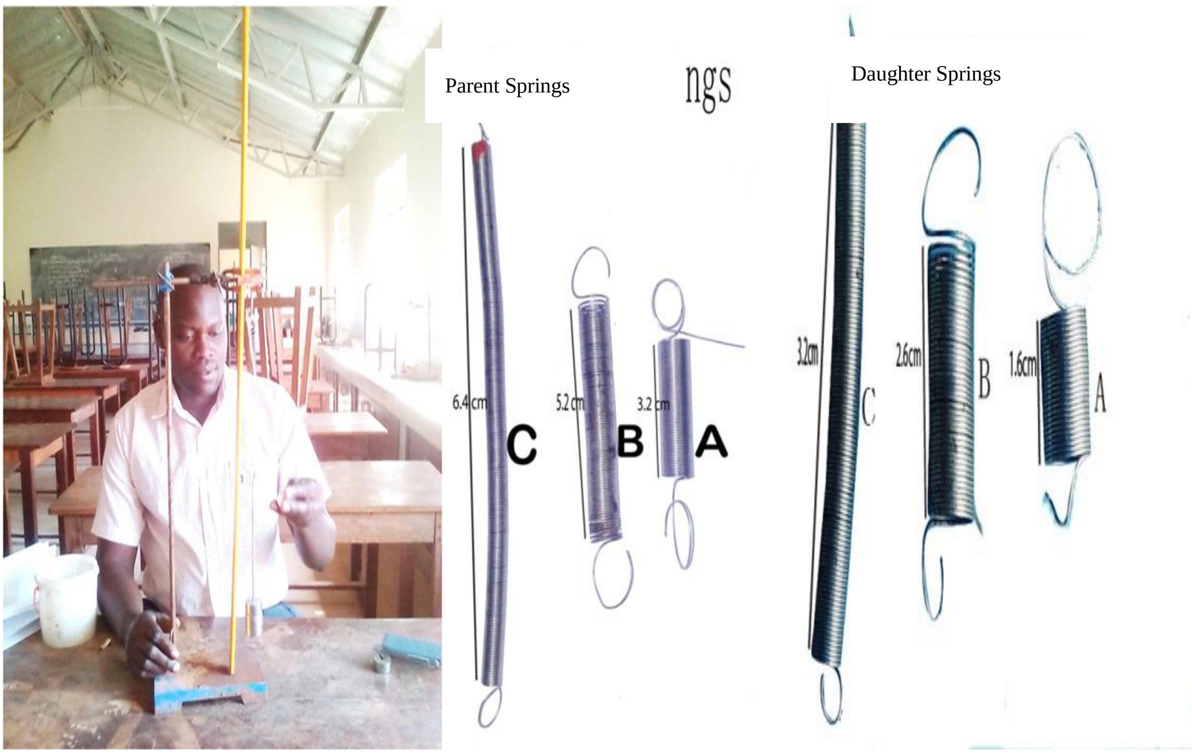

Figure 1: Laboratory demonstration of the experiments

The individual parent and daughter spring constants were calculated as,

$$k = \frac{g}{Slope} \qquad 2.1$$

Where$g$ is the acceleration due to gravity and S, the slope is calculated as,

$$Slope\ (S) = \frac{Change\ is\ extension\ (cm)}{change\ in\ mass\ (grams)} \qquad 2.2$$

3. RESULTS AND DISCUSSION

Table 2 shows the results of the achieved parent and daughter springs final pointer reading due to the added mass. The initial pointer readings were, 50.8 cm, 47.0 cm and 56.0 cm for the parent springs A, B and C respectively while as the initial pointer readings for the daughter springs were 49.5 cm, 45.5 cm and 46.2 cm for the daughter springs A, B and C respectively.

Table 2: Final pointer readings and extensions of the parent and daughter springs due to added loads

Load (g)	Parent Spring						Daughter Spring					
	Final pointer reading (cm)			Extension (cm)			Final pointer reading (cm)			Extension (cm)		
	A	B	C	A	B	C	A	B	C	A	B	C
100	53.3	56.4	58.8	2.5	9.4	2.8	50.8	50.6	48.3	1.3	5.1	2.1
200	54.7	66.1	67.0	3.9	19.1	11.0	52.2	55.6	52.5	2.7	10.1	6.3
300	58.5	76.1	75.0	7.7	29.1	19.0	63.5	60.6	56.5	4.0	15.1	10.3
400	60.1	85.8	83.1	9.3	38.8	27.1	54.6	65.4	60.6	5.1	20	14.4

Figure 2A and 2B shows the extensions of the parent's and daughter springs for the different loads.

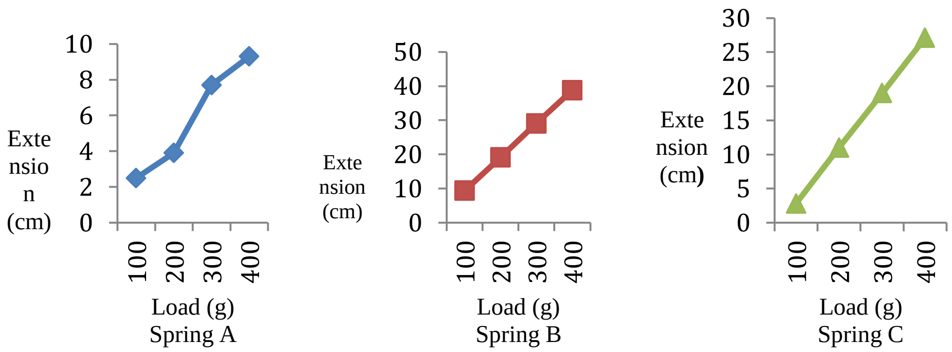

Figure 2A: Parent spring extension for the respective load

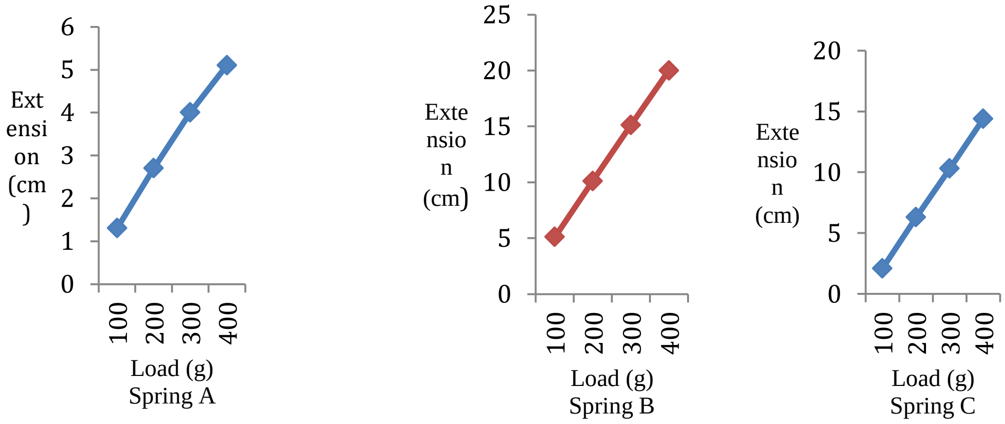

Figure 2B: Daughter spring extension for the respective load

Using figures 2A and 2B, the slopes of the respective springs, parent and daughter springs, where found using equation 2.2 and the corresponding spring constants were found using equation 2.1. Table 3 shows the slopes and spring constants for both the parent and their respective daughter springs.

Table 3: Slope and spring constants of the parent and the daughter springs.

Spring	**A**	**B**	**C**
Parent Spring Slope, S, (m/Kg)	0.224	0.971	0.799
Daughter spring Slope, S, (m/Kg)	0.133	0.500	0.410
Parent spring Constant, k, (N/m)	43.834	10.099	12.284
Daughter Spring Constant, k, (N/m)	73.815	19.620	23.944

4. CONCLUSION

In this paper, the relationship between parent and daughter spring constants have been measured and analyzed. The measurements show that there is a strong positive relationship between the values of the parent spring constant and the daughter spring constant and their values are in comparison with their magnitudes as being explained by young's equation.

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