

# Radiation – Charge Energy Transference Relations

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**Abstract:** Change in frequency of radiation is directly proportional to product of charges and inversely proportional to displacement brought from origin.

Case (i) from infinity

$$(v - v_0) \propto \frac{Qq}{x + \Delta x}$$

Case (ii) from  $x$  to  $(x + \Delta x)$

$$(v - v_0) \propto \frac{Qq(\Delta x)}{(x + \Delta x)^2}$$

**Keywords:** Radiation, directly proportional, charges.

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## 1. INTRODUCTION

Electrostatic phenomena arise from the forces that electric charges exert on each other. Coulomb's law quantifies the amount of force between two stationary electrically charged particles. The electric force between charged bodies at rest is conventionally called electrostatic force or Coulomb force<sup>1</sup>

The electrostatic potential Energy  $V_E$  of one point charge  $q$  at position  $r$  in the presence of an electric field  $E$  is defined as negative of work  $W$  done to bring it from reference position  $r_{ref}$  to that position  $r$ <sup>2</sup>

Law of conservation of energy states that the total energy of an isolated system remains constant; it is said to be conserved over time<sup>3</sup>

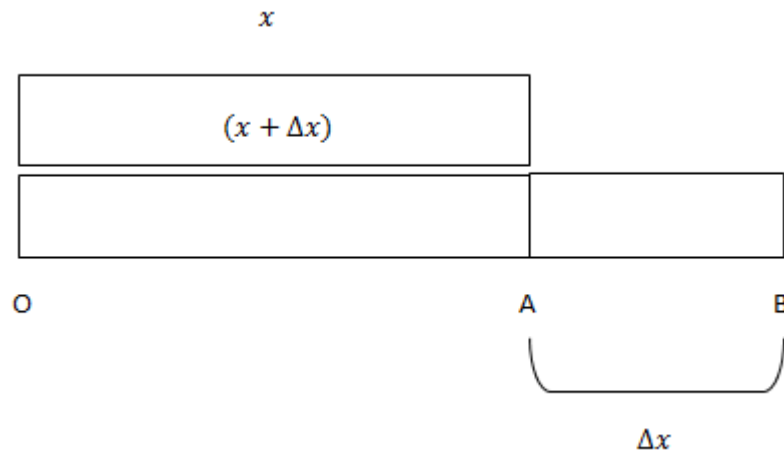
The Planck Constant, or Planck's constant is the quantum of electromagnetic action that relates a photon's energy to its frequency. The Planck constant multiplied by a photon's frequency is equal to a photon's energy<sup>4</sup>

### Assumptions:

1. Assume that a photon of energy  $h\nu$  or  $hc/\lambda$  hits a charge  $q$  having initial velocity zero attains instantaneous velocity  $v$
2. Consider charge  $Q$  at origin stationary and charge  $q$  is brought to A without accelerating.
3. Order of charge is out of range of applicable Quantum mechanics.
4. Order of mass of charge is out of range of applicable quantum mechanics
5.  $x \gg \Delta x$
6.  $Qq > 0$  if  $Qq < 0$  take  $|Qq|$  i.e. work is always done against the electric field or field causing attraction forces (if any)
7. Only electrostatic forces is experienced between charges  $Q$  and  $q$
8. Consider charge  $Q$  at origin stationary and charge  $q$  is brought to B from A without accelerating.

## 2. METHODS

[1]



$$\overline{OA} = x$$

$$\overline{AB} = \Delta x$$

$$\overline{OB} = (x + \Delta x)$$

$$\text{Let } g(x) = \int_{\infty}^x F \cdot dx$$

$$\text{And } U(x) = \text{Potential Energy at } x$$

$$= - \int_{\infty}^x F \cdot dx = \frac{+Qq}{4\pi\epsilon_0 x}$$

$$F = \text{Electrostatic force} = \frac{1}{4\pi\epsilon_0} \frac{Qq}{x^2}$$

$\therefore g(x)$  is discontinuous, it is differentiable

$$\text{Using } g(x) = \lim_{\Delta x \rightarrow 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$\Rightarrow \frac{d}{dx} g(x) = \lim_{\Delta x \rightarrow 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$\Rightarrow \frac{d}{dx} \left[ \int_{\infty}^x F \cdot dx \right] = \lim_{\Delta x \rightarrow 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$\Rightarrow F(\text{at } x + \Delta x) = \lim_{\Delta x \rightarrow 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$\Rightarrow [F_{(x+\Delta x)} \times \Delta x] = \int_{\infty}^{x+\Delta x} \frac{Qq \cdot dx}{4\pi\epsilon_0 (x+\Delta x)^2} - \int_{\infty}^x \frac{Qq \cdot dx}{4\pi\epsilon_0 (x)^2}$$

$$\Rightarrow [F_{(x+\Delta x)} \Delta x] = U_x - U_{(x+\Delta x)}$$

$$\Rightarrow -U_{(x+\Delta x)} = \frac{Qq}{4\pi\epsilon_0} \left[ \frac{\Delta x}{(x+\Delta x)^2} - \frac{1}{x} \right]$$

$$\Rightarrow -U_{(x+\Delta x)} = \frac{Qq}{4\pi\epsilon_0} \left[ \frac{x\Delta x - (x+\Delta x)^2}{x(x+\Delta x)^2} \right]$$

$$\Rightarrow -U_{(x+\Delta x)} = \frac{Qq}{4\pi\epsilon_0} \left[ \frac{x\Delta x - x^2 - 2x\Delta x}{x(x+\Delta x)^2} \right]$$

$$\Rightarrow -U_{(x+\Delta x)} = \frac{Qq}{4\pi\epsilon_0} \left[ \frac{x^2 + x\Delta x}{x(x+\Delta x)^2} \right]$$

$$\Rightarrow -U_{(x+\Delta x)} = \frac{Qq}{4\pi\epsilon_0} \left[ \frac{x(x+\Delta x)}{x(x+\Delta x)^2} \right]$$

$$\Rightarrow -U_{(x+\Delta x)} = \frac{Qq}{4\pi\epsilon_0(x+\Delta x)} \rightarrow \textcircled{2}$$

$$\Rightarrow U_{(x+\Delta x)} - U_x = -F_{(x+\Delta x)}\Delta x$$

$$\Rightarrow \Delta U = -F_{(x+\Delta x)}\Delta x \rightarrow \textcircled{1}$$

$$U_{(x+\Delta x)} = -F_{(x+\Delta x)}\Delta x + U_x$$

[2] According to assumptions ① and ②

∴ Work done is conservative

Using law of Conservative Energy

$$\Delta KE = -\Delta PE$$

$$\therefore V_{initial} = 0$$

$$V_{final} = V$$

$$\frac{1}{2}mv^2 = -[-F_{(x+\Delta x)}\Delta x + U_{(x)}] \text{ (from eq. 2)}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{Qq}{4\pi\epsilon_0(x+\Delta x)} \rightarrow \textcircled{3}$$

$$\Rightarrow V_{max} = \sqrt{\frac{2Qq}{4\pi\epsilon_0(x+\Delta x)m}}$$

Using conservation of energy law

$$\Delta KE = \Delta h\nu$$

Where  $h\nu$  = energy of radiation

Case (i) when collision is elastic

$$\Rightarrow KE = h\nu$$

$$\Rightarrow \frac{1}{2}mv^2 = h\nu$$

Using Equation ③

$$\frac{Qq}{4\pi\epsilon_0(x+\Delta x)} = h\nu$$

$$\Rightarrow v \propto \left( \frac{Qq}{x+\Delta x} \right) \text{ or } v = \left( \frac{Qq}{4\pi\epsilon_0 h(x+\Delta x)} \right)$$

From assumption ① and equation ③

$$\frac{1}{\lambda} \propto \frac{Qq}{(x+\Delta x)} \text{ or } \frac{1}{\lambda} = \frac{Qq}{4\pi\epsilon_0(x+\Delta x)hc}$$

$$\text{and } v \propto \sqrt{\frac{v}{m}} \text{ or } v = \sqrt{\frac{2h\nu}{m}}$$

Case (ii) if collision is inelastic

$$\Rightarrow \Delta KE = \Delta h\nu$$

$$\Rightarrow KE = h(\nu - \nu_0)$$

Where  $h\nu_0$  = energy absorbed

$\nu_0$  = frequency of radiation absorbed

$$\Rightarrow h(\nu - \nu_0) = \frac{1}{2}mv^2$$

$$\Rightarrow v\alpha \sqrt{\frac{\nu - \nu_0}{m}} \text{ or } v\alpha \sqrt{\frac{2h(\nu - \nu_0)}{m}}$$

From equation ③

$$\frac{Qq}{4\pi\epsilon_0(x + \Delta x)} = h(\nu - \nu_0)$$

$$\Rightarrow (\nu - \nu_0)\alpha \frac{Qq}{(x + \Delta x)} \text{ or } (\nu - \nu_0) = \frac{Qq}{4\pi\epsilon_0(x + \Delta x)h}$$

From assumption ① and equation ③

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{Qq}{4\pi\epsilon_0(x + \Delta x)}$$

$$\Rightarrow \frac{1}{\lambda} - \frac{1}{\lambda_0} \alpha \frac{Qq}{(x + \Delta x)} \text{ or } \frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{Qq}{4\pi\epsilon_0(x + \Delta x)hc}$$

Where  $\lambda_0$  = wavelength of radiation absorbed

$\frac{hc}{\lambda_0}$  = energy of radiation absorbed

[3] According to assumptions ⑧ and ①

∴ Work done is conservative

Using law of conservation of Energy

$$\Delta KE = \Delta PF$$

$$\frac{1}{2}mv^2 = -[-F_{(x+\Delta x)}\Delta x] \quad \text{equation ①}$$

$$\frac{1}{2}mv^2 = \frac{Qq(\Delta x)}{4\pi\epsilon_0(x + \Delta x)^2} \quad \rightarrow \text{④}$$

$$\Rightarrow v\alpha \sqrt{\frac{Qq\Delta x}{m(x + \Delta x)^2}} \text{ or } V_{max} = \sqrt{\frac{2Qq(\Delta x)}{m4\pi\epsilon_0(x + \Delta x)^2}}$$

Using Law of Conservation of Energy

$$\Delta KE = \Delta h\nu$$

Case (i) when collision is elastic

$$\Rightarrow \frac{1}{2}mv^2 = h\nu$$

From equation ④

$$h\nu = \sqrt{\frac{2Qq(\Delta x)}{4\pi\epsilon_0(x + \Delta x)^2}}$$

$$\Rightarrow v\alpha \frac{Qq(\Delta x)}{(x + \Delta x)^2} \text{ or } v = \frac{Qq(\Delta x)}{4\pi\epsilon_0h(x + \Delta x)^2}$$

$$v\alpha\sqrt{\frac{\gamma}{m}} \text{ or } v = \sqrt{\frac{2h\gamma}{m}}$$

From assumption ① and equation ④

$$h\gamma = \frac{hc}{\lambda} \Rightarrow \frac{hc}{\lambda} = \frac{Qq(\Delta x)}{4\pi\epsilon_0(x + \Delta x)^2}$$

$$\Rightarrow \frac{1}{\lambda} \alpha \frac{Qq(\Delta x)}{(x + \Delta x)^2} \text{ or } \frac{1}{\lambda} = \frac{Qq(\Delta x)}{4\pi\epsilon_0 hc(x + \Delta x)^2}$$

Case (ii) when collision is inelastic

$$\frac{1}{2}mv^2 = h(\gamma - \gamma_0)$$

$$\Rightarrow v\alpha\sqrt{\frac{\gamma - \gamma_0}{m}} \text{ or } v = \sqrt{\frac{2h(\gamma - \gamma_0)}{m}}$$

From equation ④

$$(\gamma - \gamma_0) = \frac{Qq(\Delta x)}{4\pi\epsilon_0 h(x + \Delta x)^2}$$

$$(\gamma - \gamma_0)\alpha \frac{Qq(\Delta x)}{(x + \Delta x)^2} \text{ or } (\gamma - \gamma_0) = \frac{Qq(\Delta x)}{4\pi\epsilon_0 h(x + \Delta x)^2}$$

$$\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \alpha \frac{Qq(\Delta x)}{(x + \Delta x)^2} \text{ or } \left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = \frac{Qq(\Delta x)}{4\pi\epsilon_0 hc(x + \Delta x)^2}$$

### 3. RESULT

1. Change in reciprocal wavelength ( $\lambda^{-1}$ ) or frequency ( $\nu$ ) of radiation is directly proportional to product of charges and inversely proportional to distance from origin

(i) From infinity:  $\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \alpha \frac{Qq}{x + \Delta x}$

(ii) From  $x$  to  $(x + \Delta x)$ :  $\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \alpha \frac{Qq\Delta x}{(x + \Delta x)^2}$

2. Maximum velocity that could be attained is directly proportional to square root of product of mass of charge and distance from origin

(i) From infinity:  $V_{max} \alpha \sqrt{\frac{Qq}{m(x + \Delta x)}}$

(ii) From  $x$  to  $(x + \Delta x)$ :  $V_{max} \alpha \sqrt{\frac{Qq(\Delta x)}{m(x + \Delta x)^2}}$

(iii) Potential Energy difference between two points is equal to electrostatic force applied at final point into displacement brought.

### APPENDIX

$\gamma$ : frequency of radiation

$\gamma_0$ : frequency of radiation absorbed

$\lambda$ : wavelength of radiation

$\lambda_0$ : wavelength of radiation absorbed

$v$ : velocity of charge  $q$

$m$ : mass of charge  $q$

$Q$ : Stationary charge value

$q$ : Value of charge undergoing interactions

$$h: \text{Planck's constant} = 6.626 \times 10^{-34} \text{ J/s}$$

$\epsilon_0$ : Permittivity of free space =  $8.854 \times 10^{-12} \text{ Farad/m}$

$(4\pi\epsilon_0)^{-1}$ : Constant of proportionality Coulomb Force =  $9 \times 10^9 \text{ Kgm}^3\text{s}^{-4}\text{A}^{-2}$

$c$ : Speed of light =  $3 \times 10^8 \text{ m/s}$

#### ACKNOWLEDGEMENT

I hereby acknowledge the unparalleled help and support given to me by Mr. Suresh Ghanshyam Kabra and Mr. Harsh Madan Somani in terms of making the manuscript, publishing and advices. I also would like to extend my deepest gratitude to Mrs. Nina Jani (HOD, Physics department DPS Bopal, Ahmedabad), my physics teacher, who has helped me understand and evolve my thinking in the concepts of physics.

#### REFERENCES

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- [1] Walker, Halliday and Resnick (2014); page 609-611
  - [2] Electromagnetism (2<sup>nd</sup> Edition), I.S. Grant, W.R. Phillips, Manchester Physics series, 2008 ISBN
  - [3] Richard Feynman (1970), the Feynman Lectures on Physics Vol. I. Addison Wesley. ISBN 978-0-201-02115-8
  - [4] Planck, Max 1909 "On the Law of Distribution of Energy in the Normal Spectrum)
  - [5] Page 161 NCERT Maths Textbook Part I Class XII