ISSN 2348-1196 (print)
International Journal of Computer Science and Information Technology Research ISSN 2348-120X (online)
Vol. 8, Issue 4, pp: (76-79), Month: October - December 2020, Available at: www.researchpublish.com

Application of Maple: Taking Some Problems as Examples

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Abstract: **In this paper, we use Maple to study the limit problem, the differential problem and the integral problem. Moreover, three examples are proposed to demonstrate the calculations. The research method adopted in this study is to find the solutions through manual calculations and verify these answers by using Maple.**

Keywords: **Maple, Limit problem, Differential problem, Integral problem.**

I. INTRODUCTION

The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, we can refer to [1-5].

This paper uses Maple as an auxiliary tool in mathematics teaching and research. In calculus and engineering mathematics courses, there are many topics involve solving the limit problems, differential problems and integral problems with Maple [6-15]. In this article, we study the following three problems:

1.1 Find the limit

$$lim_{x\to 0}\left(\frac{a_1^x+a_2^x+\cdots+a_n^x}{n}\right)^{\frac{1}{x}}, \tag{1}$$

where $a_1, a_2, \cdots, a_n > 0$.

1.2 If $f(x) = x^2 sin2x$, find the 50-th order derivative of $f(x)$,

$$f^{(50)}(x). \tag{2}$$

1.3 Find the indefinite integral

$$\int \frac{1}{x+\sqrt{1-x^2}}dx. \tag{3}$$

II. METHODS AND MATERIALS

Firstly, we introduce some notations, formulas and theorem used in this article.

2.1 Notations:

2.1.1 We introduce some instructions of Maple:

>f:=x->x^2*sin(2*x);

Define $f(x) = x^2 sin2x$.

>limit(f(x),x=0);

Find the value of $lim_{x\to 0} f(x)$.

>((D@@50)(f))(Pi/2);

Find the value of $f^{(50)}\left(\frac{\pi}{2}\right)$.

>int(f(x),x=a..b);

Find the definite integral of function $f(x)$ from $x = a$ to $= b$.

2.1.2 $lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$.

2.1.3 Leibniz rule: If u, v are C^n functions of variable x. Then

$$(uv)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} u^{(k)}(x) v^{(n-k)}(x), \tag{4}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

III. RESULTS AND DISCUSSIONS

In the following, we solve the three problems discussed in this paper. Moreover, Maple is used to calculate some approximations to verify our answers.

Problem 3.1: The limit problem

$$\lim_{x\to 0} \left(\frac{a_1^x + a_2^x + \cdots + a_n^x}{n}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\left[\left(1 + \frac{a_1^x + a_2^x + \cdots + a_n^x - n}{n}\right)^{\frac{n}{a_1^x + a_2^x + \cdots + a_n^x - n}}\right]^{\frac{a_1^x + a_2^x + \cdots + a_n^x - n}{nx}}$$

$$= \exp\left(\lim_{x\to 0}\frac{a_1^x + a_2^x + \cdots + a_n^x - n}{nx}\right)$$

$$= \exp\left(\lim_{x\to 0}\frac{a_1^x lna_1 + a_2^x lna_2 + \cdots + a_n^x lna_n}{n}\right)$$

$$= \exp\left(\frac{ln(a_1 \cdot a_2 \cdots a_n)}{n}\right)$$

$$= \sqrt[n]{a_1 \cdot a_2 \cdots a_n}. \tag{5}$$

Therefore, we can obtain

$$\lim_{x\to 0}\left(\frac{2^x + 5^x + 7^x}{3}\right)^{\frac{1}{x}} = \sqrt[3]{70}\,. \tag{6}$$

Next, we use Maple to verify the correctness of Eq. (6).

>limit(((2^x+5^x+7^x)/3)^(1/x),x=0);

$$2^{1/3} \cdot 5^{1/3} \cdot 7^{1/3}$$

Problem 3.2: The differential problem

$f(x) = x^2 sin2x$, then by Leibniz rule, the 50-th order derivative of $f(x)$ is

$$f^{(50)}(x)$$
$$=\sum_{k=0}^{50}\binom{50}{k}(x^2)^{(k)}(sin2x)^{(50-k)}$$
$$=\sum_{k=0}^{2}\binom{50}{k}(x^2)^{(k)}(sin2x)^{(50-k)}$$
$$=x^2(sin2x)^{(50)}+50\cdot 2x\cdot(sin2x)^{(49)}+1225\cdot 2\cdot(sin2x)^{(48)}$$
$$=2^{50}\cdot x^2\cdot sin\left(2x+\frac{50}{2}\pi\right)+2^{49}\cdot 100x\cdot sin\left(2x+\frac{49}{2}\pi\right)+2^{48}\cdot 2450\cdot sin\left(2x+\frac{48}{2}\pi\right)$$
$$=2^{49}\cdot(-2\cdot x^2 sin2x+100\cdot xcos2x+1225\cdot sin2x). \quad (7)$$

And hence,

$$f^{(50)}\left(\frac{\pi}{2}\right)=-25\cdot 2^{50}\cdot\pi. \quad (8)$$

In the following, Maple is used to prove the correctness of Eq. (8).

```
>f:=x->x^2*sin(2*x);
>((D@@50)(f))(Pi/2);
```

$$-28147497671065600\pi$$

```
>-25*2^50*Pi;
```

$$-28147497671065600\pi$$

Problem 3.3: The indefinite integral problem

$$\int\frac{1}{x+\sqrt{1-x^2}}dx$$
$$=\int\frac{cost}{sint+cost}dt \quad (\text{Let } x=sint, \text{ then } dx=costdt)$$
$$=\frac{1}{2}\left(\int\frac{cost-sint}{sint+cost}dt+\int 1dt\right)$$
$$=\frac{1}{2}(ln|sint+cost|+t)+C\ .$$
$$=\frac{1}{2}\left(ln\left|x+\sqrt{1-x^2}\right|+arcsinx\right)+C\ . \quad (9)$$

Thus, the definite integral

$$\int_0^1\frac{1}{x+\sqrt{1-x^2}}dx=\frac{\pi}{4}\ . \quad (10)$$

Using Maple to check our result.

```
>int(1/(x+(1-x^2)^(1/2)),x=0..1);
```

$$\frac{\pi}{4}$$

IV. CONCLUSION

In this article, we provide some techniques to evaluate the limit problem, the differential problem and the integral problem. We hope these techniques can be applied to solve another similar problems. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topics to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

REFERENCES

[1] M. L. Abell and J. P. Braselton, Maple by Example, 3rd ed., Elsevier Academic Press, 2005.

[2] R. M. Corless, Essential Maple, Springer-Verlag, New York, 1994.

[3] F. Garvan, The Maple Book, Chapman & Hall/CRC, 2001.

[4] A. Heck, Introduction to Maple, 3rd ed., SpringerVerlag, New York, 2003.

[5] D. Richards, Advanced Mathematical Methods with Maple, Cambridge University Press, 2002.

[6] C. -H. Yu, "Solving some definite integrals using Parseval's theorem," American Journal of Numerical Analysis, Vol. 2, No. 2, pp. 60-64, 2014.

[7] Systems and Software, Vol. 2, No. 1, pp. 22-26, 2014.

[8] C. -H. Yu, "Using Maple to study the double integral problems," Applied and Computational Mathematics, Vol. 2, No. 2, pp. 28-31, 2013.

[9] C. -H. Yu, "Using Maple to study the partial differential problems, " Applied Mechanics and Materials, Vols. 479-480, 800-804, 2013.

[10] C. -H. Yu, "Evaluating the partial derivatives of four types of two-variables functions, " Automatic Control and Information Sciences, Vol. 2, Issue. 1, 1-6, 2014.

[11] C. -H. Yu, " Using differentiation term by term theorem to study the partial differential problems, " Turkish Journal of Analysis and Number Theory, Vol. 1, No. 1, 63-68, 2013.

[12] C. -H. Yu, " A study of limit problems, " (in Chinese), Journal of Nan Jeon, Vol. 16, D2-1~14, 2013.

[13] C. -H. Yu, " Application of Maple on the limit problems, " Universal Journal of Engineering Science, Vol. 5, No. 3, pp. 39 - 43, 2017.

[14] C. -H. Yu, " Evaluating the Derivatives of Two Types of Functions," International Journal of Computer Science and Mobile Computing , Vol. 2, Issue. 7, pp. 108-113, 2013.

[15] C. -H. Yu, "Using Maple to Evaluate the Derivatives of Some Functions," International Journal of Research in Computer Applications and Robotics, Vol. 1, Issue. 4, pp. 23-31, 2013.